1. Dado $\mathbf{u}=(1,2,0)$ y $\mathbf{v}=(0,2,1)$ calcula un vector $\mathbf{w}$ tal que $\mathbf{w}\perp\mathbf{u}$, $\mathbf{w}\perp\mathbf{v}$, e $\|\mathbf{w}\|=3$

2. Hallar a posición relativa das rectas $r_1: x=\frac{y-1}{2}=\frac{z-2}{3}$ e $r_2: \frac{x-3}{-4}=\frac{y-2}{-3}=\frac{z-1}{2}$

3. Atopa un intervalo $[a,b]$ e unha función y $g([a,b])\subset\mathbb{R}\to\mathbb{R}^2$ que verifique que $g([a,b])$ é semielipse da ecuación $(x-1)^2+4y^2=1$ con $y\geq 0$

$[a,b]=$ $\qquad\qquad$ $g(t)$

4. Escribe **E,P,H**

a) $9(x-2)^2-(y+3)^2=36$

b) $x^2+2x-4y-3=0$

c) $9(x-4)^2+16(y-3)^2=144$

d) $25x^2+9y^2-100x+54y-4=0$

5. Escribir N (se non ten solución); E(se é un escalar) ou V(se é vectorial)

a) $(\mathbf{u}\ \mathbf{x}\ \mathbf{v})\cdot\mathbf{w}$

b) $\lambda\mathbf{u}\ \mathbf{x}(\mathbf{uv}\ \mathbf{x}\ \mathbf{w})$

c) $\lambda\mathbf{u}\ \mathbf{x}(\mathbf{uv}-\mathbf{w})$

d) $\lambda\mathbf{u}+(\mathbf{v}\ \mathbf{x}\ \mu\mathbf{w})$

6. Describe e fai un esbozo de $-16x^2+y^2+16z^2=4$

7. Dominio de:

a) $f(x,y)=\sqrt[3]{\frac{x+2}{\operatorname{sen} y}}$

b) $f(x,y)=\sqrt{x+y}$

c) $f(x,y)=\dfrac{\log\left(x^2+y^2-9\right)}{x-1}$

d) $f(x,y,z)=e^{y/x}+e^{z/x}$

8. Dominio interior e fronteira

9. Calcula ecuación planos tanxentes $Ax+By+Cz+D=0$

$f(x,y)=\sqrt{x^2+y^2}$ en $(0,0,0)$ e $\left(1,1,\sqrt{2}\right)$

10. $f(x,y)=x\ \text{sen}\dfrac{1}{xy}$ $\qquad g(x,y)=\dfrac{xy}{x^2+2y^2}$

Límites $\displaystyle \lim_{\substack{x\to 0\\ y=mx}} g(x,y)$

12. Calcula e clasifica os puntos críticos de $f(x,y)=9x+3x^2-x^3-3y^2+y^3$

13. Atopa os extremos absolutos de $f(x,y)=9x+3x^2-x^3-3y^2+y^3$ en $S=\left\{(x,y)\in\mathbb{R}^2 \,/\, x^2+2y^2=6\right\}$

SOLUCIONES

1.

$\mathbf{u}=(1,2,0)$ y $\mathbf{v}=(0,2,1)$

$$\begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix} = 2\vec{\mathbf{i}}+2\vec{\mathbf{k}}-\left(\vec{\mathbf{j}}\right)=2\vec{\mathbf{i}}+2\vec{\mathbf{k}}=(2,-1,2)$$

$$\left\|(2,-1,2)\right\|=\sqrt{4+1+4}=\sqrt{9}=3$$

Por tanto, basta tomar $\mathbf{w}=(2,-1,2)$ o $\mathbf{w}=(-2,1,-2)$

2.

$$\mathbf{r}_1:\mathbf{x}=\frac{\mathbf{y}-1}{2}=\frac{\mathbf{z}-2}{3}$$

$$\mathbf{r}_2:\frac{\mathbf{x}-3}{-4}=\frac{\mathbf{y}-2}{-3}=\frac{\mathbf{z}-1}{2}$$

$$\left.\begin{array}{l}\mathbf{v}_{\mathbf{r}_1}=(1,2,3)\\ \mathbf{v}_{\mathbf{r}_2}=(-4,-3,2)\end{array}\right\}\Rightarrow$$ Os vectores non son proporcionais, polo tanto, non son nin paralelos ni coincidentes.

$\mathbf{r}_1$ e $\mathbf{r}_2$ son secantes ou crúzanse.

$$\begin{vmatrix} 1 & -4 & -3 \\ 2 & -3 & -1 \\ 3 & 2 & 1 \end{vmatrix} = -3\cancel{-12}\cancel{+12}-(27-2-8)=-20\neq 0$$

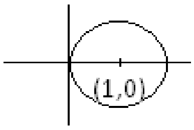
$$\mathbf{P}_1-\mathbf{P}_2=(0,1,2)-(3,2,1)=(-3,-1,1)$$

Por tanto, as rectas crúzanse.

3.

$$(x-1)^2+4y^2=1 \quad y\geq 0$$

$$\left.\begin{array}{r} x-1=2\cos t \\ y=\operatorname{sen} t \end{array}\right\} \Rightarrow \begin{array}{r} x=1+2\cos t \\ y=\operatorname{sen} t \end{array}$$



$$t\in[0,\pi]\ y\geq 0$$

$$g(t)=(1+2\cos t,\operatorname{sen} t) \qquad [a,b]=[0,\pi]$$

4.

a) $9(x-2)^2-4(y+3)^2=36$

$\dfrac{(x-2)^2}{4}-\dfrac{(y+3)^2}{9}=1$. Hipérbola

b) $x^2+2x-4y-3=0$

$(x+1)^2=x^2+2x+1$

$x^2+2x-4y-3=(x+1)^2-1-4y-3=(x+1)^2-4y-4=0\Rightarrow$

$4(y+1)=(x+1)^2$. Parábola.

c) $9(x-4)^2+16(y-3)^2=144$

$\dfrac{(x-4)^2}{16}+\dfrac{(y-3)^2}{9}=1\Rightarrow$. Elipse.

d) $25\mathbf{x}^2+9\mathbf{y}^2-100\mathbf{x}+54\mathbf{y}-4=0$

$$(5\mathbf{x}-10)^2=25\mathbf{x}^2-100\mathbf{x}+100$$
$$+\quad(3\mathbf{y}+9)^2=9\mathbf{y}^2+54\mathbf{y}+81$$

$$(5\mathbf{x}-10)^2+(3\mathbf{y}+9)^2=25\mathbf{x}^2-100\mathbf{x}+9\mathbf{y}^2+54\mathbf{y}+181$$

$(5\mathbf{x}-10)^2+(3\mathbf{y}+9)^2=185$. Elipse.

5.

a) $(\mathbf{u}\ \mathbf{x}\ \mathbf{v})\cdot\mathbf{w}$ .Vector

b) $\lambda\mathbf{u}(\mu\mathbf{v}\ \mathbf{x}\,\mathbf{w})$. Escalar

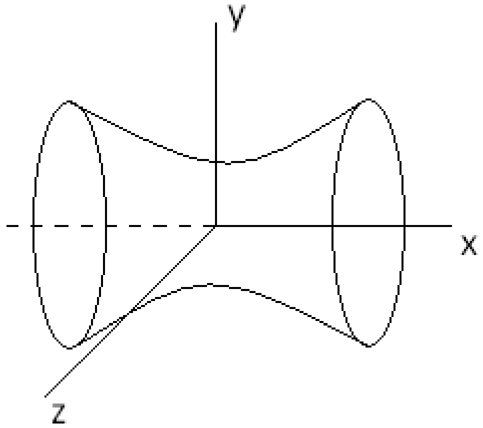
c) $\lambda\mu\ \mathbf{x}\ (\mu\mathbf{v}-\mathbf{w})$. Vector

d) $\lambda\mathbf{u}+(\mathbf{v}\ \mathbf{x}\,\mu\mathbf{w})$. Vector

6.

$$-16\mathbf{x}^2+\mathbf{y}^2+16\mathbf{z}^2=4\Rightarrow-4\mathbf{x}^2+\frac{\mathbf{y}^2}{4}+4\mathbf{z}^2=1\Rightarrow$$

$\Rightarrow-\dfrac{\mathbf{x}^2}{1/4}+\dfrac{\mathbf{y}^2}{4}+\dfrac{\mathbf{z}^2}{1/4}=1$. Hiperboloide de una hoja.



7.

a) $\mathbf{f}(\mathbf{x},\mathbf{y})=\sqrt[3]{\frac{\mathbf{x}+2}{\mathbf{seny}}}$

$\mathbf{sen\ y}=0\Leftrightarrow\mathbf{y}=\mathbf{k}\pi$ con $\mathbf{k}\in\mathbb{Z}$

Entonces $\mathbf{Dom\ f}=\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\ /\ \mathbf{y}\neq\mathbf{k}\pi,\mathbf{k}\in\mathbb{Z}\right\}$

b) $\mathbf{f}(\mathbf{x},\mathbf{y})=\sqrt{\mathbf{x}+\mathbf{y}}$

$\mathbf{x}+\mathbf{y}\geq0\Leftrightarrow\mathbf{y}\geq-\mathbf{x}$

$\mathbf{Dom\ f}=\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\ /\ \mathbf{y}\geq-\mathbf{x}\right\}$

c) $\mathbf{f}(\mathbf{x},\mathbf{y})=\frac{\log\left(\mathbf{x}^2+\mathbf{y}^2-4\right)}{\mathbf{x}-1}$

$\mathbf{x}^2+\mathbf{y}^2-4>0\Rightarrow\mathbf{x}^2+\mathbf{y}^2>4$

$\mathbf{x}-1=0\Leftrightarrow\mathbf{x}=1$

$\mathbf{Dom\ f}=\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\ /\ \mathbf{x}\neq1,\mathbf{x}^2+\mathbf{y}^2>4\right\}$

d) $\mathbf{f}(\mathbf{x},\mathbf{y},\mathbf{z})=\mathbf{e}^{\mathbf{y}/\mathbf{x}}+\mathbf{e}^{\mathbf{z}/\mathbf{x}}$

$\mathbf{Dom\ f}=\left\{(\mathbf{x},\mathbf{y},\mathbf{z})\in\mathbb{R}^3\ /\ \mathbf{x}\neq0\right\}$

9.

$\mathbf{f}(\mathbf{x},\mathbf{y})=\sqrt{\mathbf{x}^2+\mathbf{y}^2}$ en $(0,0,0)$ e $\left(1,1,\sqrt{2}\right)$

$$\frac{\partial\mathbf{f}}{\partial\mathbf{x}}(0,0)=\lim_{\mathbf{h}\to0}\frac{\mathbf{f}(\mathbf{h},0)-\mathbf{f}(0,0)}{\mathbf{h}}=\lim_{\mathbf{h}\to0}\frac{|\mathbf{h}|-0}{\mathbf{h}}\nexists$$

$$\frac{\partial\mathbf{f}}{\partial\mathbf{y}}(0,0)=\lim_{\mathbf{h}\to0}\frac{\mathbf{f}(0,\mathbf{h})-\mathbf{f}(0,0)}{\mathbf{h}}=\lim_{\mathbf{h}\to0}\frac{|\mathbf{h}|-0}{\mathbf{h}}\nexists$$

Polo tanto, non existe plano tanxente no $(0,0,0)$.

$$\frac{\partial f}{\partial x}=\frac{x}{\sqrt{x^2+y^2}} \qquad \frac{\partial f}{\partial y}=\frac{y}{\sqrt{x^2+y^2}}$$

$$z-\sqrt{2}=\frac{1}{2}(x-1)+\frac{1}{2}(y-1)$$

10.

$$\lim_{\substack{x\to 0\\ y=mx}} g(x,y)=\lim_{\substack{x\to 0\\ y=mx}}\frac{xy}{x^2+2y^2}=\lim_{x\to 0}\frac{m\cancel{x^2}}{\cancel{x^2}+2m^2\cancel{x^2}}=\frac{m}{1+2m^2}$$

11. $f(x,y)=9x+3x^2-x^3-3y^2+y^3$

$$\frac{\partial f}{\partial x}=9+6x-3x^2=0\Leftrightarrow\begin{cases}x=-1\\ x=3\end{cases}$$

$$\frac{\partial f}{\partial y}=-6y+3y^2=0\Leftrightarrow\begin{cases}y=0\\ y=2\end{cases}$$

Por tanto, os puntos críticos da función son (-1,0), (-1,2),(3,0),(3,2).

$$\frac{\partial^2 f}{\partial x^2}=6-6x$$

$$\frac{\partial^2 f}{\partial y^2}=-6+6y$$

$$\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}=0$$

$$H(x,y)=\begin{vmatrix}6-6x & 0\\ 0 & -6+6y\end{vmatrix}$$

Por tanto:

$H(-1,0)=\begin{vmatrix}12 & 0\\ 0 & -6\end{vmatrix}<0$ O punto (-1,0) é un punto de sela.

$H(-1,2)=\begin{vmatrix}12 & 0\\ 0 & 6\end{vmatrix}>0$ En (-1,2) hai un mínimo relativo.

$H(3,0)=\begin{vmatrix}-12 & 0\\ 0 & -6\end{vmatrix}>0$ En (3,0) hai un máximo relativo.

$H(3,2)=\begin{vmatrix}-12 & 0\\ 0 & 6\end{vmatrix}<0$ En (3,2) hai un punto de sela.

12. $f(x,y)=9x+3x^2-x^3-3y^2+y^3$ en $x^2+2y^2=6$

$$\begin{cases} 9+6x-3x^2=2x\lambda \\ -6y+3y^2=4y\lambda \\ x^2+2y^2=6 \end{cases}$$

$$18y+12xy-6x^2y=4xy\lambda \Rightarrow -6y(x-3)(x+1)=4xy\lambda$$

$$-6xy+3xy^2=4xy\lambda \Rightarrow y(-6x+3xy)=4xy\lambda$$

$$x=\frac{-6\pm\sqrt{36-4\cdot(-3)\cdot 9}}{-6}=\frac{-6\pm 12}{-6} \begin{cases} \nearrow x=3 \\ \searrow x=-1 \end{cases}$$

$$-6y(x-3)(x+1)=3yx(-2+y) \rightarrow \begin{cases} \nearrow y=2,\ x=3 \quad \text{X} \\ y=2,\ x=-1 \quad \text{X} \\ \searrow y=0,\ x=\pm\sqrt{6} \end{cases}$$

$$x^2+2y^2=6$$
$$9+2\cdot 4\neq 6$$
$$1+2\cdot 4\neq 6$$

Pode haber mais solución, hai que resolver o sistema:

$$\begin{cases} -6y(x-3)(x+1)=3yx(-2+y) \\ x^2+2y^2=6 \end{cases}$$

Resolvendo o sistema con wolfram alpha obtemos a maiores as solucións:

$$x=-\frac{2}{3},\ y=-\frac{5}{3}$$

$$x=2-\frac{2^{2/3}}{\sqrt[3]{2-\sqrt{2}}}-\sqrt[3]{2(2-\sqrt{2})}\approx -0.95137,\ y\approx 1.59607$$

Por tanto, temos os puntos $(\sqrt{6},0),(-\sqrt{6},0),\left(-\frac{2}{3},-\frac{5}{3}\right)$ e $(-0.95137,1.59607)$.

Evaluando a función nos puntos anteriores temos que:

$$f(\sqrt{6},0)=25.348$$

$$f(-\sqrt{6},0)=10.65153$$

$$f\left(-\frac{2}{3},-\frac{5}{3}\right)=-17.34$$

$$f(-0.95137,1.59607)=-8.5623$$

Por tanto, f alcanza o máximo absoluto no punto $(\sqrt{6},0)$ e o mínimo absoluto no punto $\left(-\frac{2}{3},-\frac{5}{3}\right)$.