TIPO B

1. $h(x,y) = \sqrt{\ln(y^2 - x^2)}$

$\ln\left(y^2 - x^2\right) \geq 0 \Rightarrow y^2 - x^2 \geq 1,$ por tanto:

Dom (h) = $\left\{\left(x,y\right) \in \mathbb{R}^2 \;/\; y^2 - x^2 \geq 1 \right\}$

2. *C* = $\left\{\left(x,y\right) \in \mathbb{R}^2 / x^2 < y^2\right\} = \left\{\left(x,y\right) \in \mathbb{R}^2 / x^2 - y^2 < 0\right\} = \left\{\left(x,y\right) \in \mathbb{R}^2 / \left(x-y\right)\left(x+y\right) < 0\right\} =$

$$\left(x-y\right)\left(x+y\right) < 0 \Rightarrow \begin{cases} x-y<0,\; x+y<0 \\ \text{ou ben} \\ x-y<0, x+y<0 \end{cases} \Rightarrow \begin{cases} y>x, y<-x \\ \text{ou ben} \\ y<x, y>-x \end{cases}$$

$$C = \left\{\left(x,y\right) \in \mathbb{R}^2 / y > x,\; y < -x\right\} \cup \left\{y < x,\; y > -x\right\}$$

Por tanto:

$$Int(C) = C \quad (C \;\text{é aberto})$$

$$Fr(C) = \left\{\left(x,y\right) \in \mathbb{R}^2 / y = x\right\} \cup \left\{\left(x,y\right) \in \mathbb{R}^2 / y = -x\right\}$$

3. $f(x,y) = -xe^{\frac{-x^2-y^2}{2}}$

$$\frac{\partial f}{\partial x} = -e^{\frac{-x^2-y^2}{2}} + x^2 e^{\frac{-x^2-y^2}{2}} = 0 \Leftrightarrow x = \pm 1$$

$$\frac{\partial f}{\partial y} = -xe^{\frac{-x^2-y^2}{2}}(-y) = xye^{\frac{-x^2-y^2}{2}} = 0 \Leftrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

Por tanto, os puntos críticos son:

(1,0) , (-1, 0)

4. $f(x,y) = x^2 y \qquad D = \left\{\left(x,y\right) \in \mathbb{R}^2 / x^2 + 2y^2 = 6\right\}$

$$\nabla f(x,y) = (2xy, x^2)$$
$$\nabla g(x,y) = (2x, 4y)$$

$$\begin{cases} 2xy = 2\lambda x \Leftrightarrow 2x(y-\lambda)=0 \Leftrightarrow \begin{cases} x=0 \\ y=\lambda \end{cases} \\ x^2 = 4y\lambda \\ x^2+2y^2=6 \end{cases}$$

- **x=0**

$$x^2+2y^2=6 \Rightarrow 2y^2=6 \Rightarrow y^2=3 \Rightarrow y=\pm\sqrt{3} \quad (0,\sqrt{3})\ ,\ (0,-\sqrt{3})$$

- **y =** λ

$$x^2 = 4\lambda^2 \Rightarrow x^2 = 4\lambda^2 \Rightarrow x = \pm 2\lambda$$

$$4\lambda^2 + 2\lambda^2 = 6 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

$$\lambda = 1 \Rightarrow y = 1 \Rightarrow \begin{cases} x=2 \\ x=-2 \end{cases} \qquad (2,1) \quad , \quad (-2,1)$$

$$\lambda = -1 \Rightarrow y = -1 \Rightarrow \begin{cases} x=2 \\ x=-2 \end{cases} \qquad (2,-1) \quad , \quad (-2,-1)$$

$$f(0,\sqrt{3}) = 0$$

$$f(0,-\sqrt{3}) = 0$$

$$\left.\begin{array}{l} f(2,-1) = -4 \\ f(-2,-1) = -4 \end{array}\right\} \textit{Mínimo absoluto}$$

$$\left.\begin{array}{l} f(2,1) = 4 \\ f(-2,1) = 4 \end{array}\right\} \textit{Máximo absoluto}$$

5. $x^2-2x+y^2-z^2-4z-3=0$

$$(x-1)^2+y^2-(z+2)^2=x^2-2x+1+y^2-(z^2+4+4z)=x^2-2x+y^2-z^2-4z-3=0$$

Por tanto:

$$(x-1)^2+y^2-(z+2)^2=0\Rightarrow(z+2)^2=(x-1)^2+y^2$$

Cono de vértice no (1 , 0 , -2)

$$x^2-2x+y^2+z^2+4z+5=0$$

$$(x-1)^2+y^2+(z+2)^2=x^2-2x+1+y^2+z^2+4z+4=x^2-2x+y^2+z^2+4z+5=0$$

$$(x-1)^2+y^2+(z+2)^2=0\Rightarrow\begin{cases}x-1=0\Rightarrow x=1\\ y=0\\ z+2=0\Rightarrow z=-2\end{cases}$$

É un punto: (1 , 0 , -2)

6. $A=\int_{-2}^{0}(-x-(-x-1))dx=\int_{-2}^{0}dx=x]_{-2}^{0}=2u^2$

7.

$$\int_{-4}^{0}\int_{-\frac{x}{2}}^{2} f(x,y)dydx$$

$$\begin{cases}-4<x<0\\ -\frac{x}{2}<y<2\end{cases}$$

$$\begin{cases}0<y<2\\ -2y<x<0\end{cases}$$

$$\int_{0}^{2}\int_{-2y}^{0} f(x,y)dxdy$$

8. Escribir en coordenadas polares e resolve a seguinte integral onde a é un número real estrictamente positivo.

$$\int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}}\sqrt{a^2-x^2-y^2}\,dxdy=\int_{0}^{a}\int_{0}^{2\pi}\rho\sqrt{a^2-\rho^2}\,d\theta d\rho$$

$$\begin{cases}x=\rho cos\theta\\ y=\rho sen\theta\end{cases}$$

$$\left.\begin{array}{l}x=\pm\sqrt{a^2-y^2}\Rightarrow x^2+y^2=a^2\\ -a\le y\le a\end{array}\right\}\Rightarrow \rho\in[0,a],\theta\in[0,2\pi]$$

9. $S=\{(x,y,z)\in\mathbb{R}^2/(x-1)^2+y^2\le(z+2)^2,\ (x-1)^2+y^2+(z+2)^2\le 2,\ z\le-2\}$

$$(z+2)^2\ge(x-1)^2+y^2\Rightarrow(z+2)\ge\pm\sqrt{(x-1)^2+y^2}\Rightarrow z\ge-2\pm\sqrt{(x-1)^2+y^2}$$

$$(z+2)^2\le 2-(x-1)^2-y^2\Rightarrow(z+2)\le\pm\sqrt{2-(x-1)^2-y^2}\Rightarrow z\le-2\pm\sqrt{2-(x-1)^2-y^2}$$

Esfera centrada en (1 , 0 , -2), cono con vértice en (1 , 0 , -2). Hallamos a intersección de esfera e como:

$$\left.\begin{array}{l}(z+2)^2=(x-1)^2+y^2\\ \\ (z+2)^2=2-(x-1)^2-y^2\end{array}\right\}\Rightarrow (x-1)^2+y^2=2-(x-1)^2-y^2$$

$$2(x-1)^2+2y^2=2\Rightarrow (x-1)^2+y^2=1$$

Por tanto:

$0\le x\le 2$

$-\sqrt{1-(x-1)^2}\le y\le \sqrt{1-(x-1)^2}$

$x\in[0,2]$

$y^2=1-(x-1)^2\Rightarrow y=\pm\sqrt{1-(x-1)^2}$

$E\ z,$

$z=-2\pm\sqrt{(x-1)^2+y^2}$

$z=-2\pm\sqrt{1-(x-1)^2-y^2}$

Por tanto:

$$V=\int_0^2\int_{-\sqrt{1-(x-1)^2}}^{\sqrt{1-(x-1)^2}}\int_{-2-\sqrt{1-(x-1)^2-y^2}}^{-2-\sqrt{(x-1)^2+y^2}} dzdydx$$

10. Hallamos o plano que pasa por os puntos (0 , 0, 2) , (3 , 2 , 0) e (9 , 0 , 0).

(3 , 2 , 0) – (0 , 0 , 2) = (3 , 2 , -2)

(9, 0 , 0) – (0 , 0 , 2) = (9 , 0 , -2)

$$\pi:\begin{vmatrix} x & y & z-2\\ 3 & 2 & -2\\ 9 & 0 & -2\end{vmatrix}=-4x-12y-18z+36=0\Rightarrow \pi:2x+6y+9z-18=0$$

Calculamos a proyección sobre XY:

$$z=0 \Rightarrow 2x+6y-18=0 \Rightarrow x+3y-9=0$$

Calculamos a recta que pasa por os puntos (0,0) e (3,2):

Un vector de esta recta será (3,2) e un punto o (0,0):

$$\frac{x}{3}=\frac{y}{2} \Rightarrow y=\frac{2x}{3}$$

Polo tanto, a rexión: $0 \le y \le 2, \frac{3y}{2} < x < 9-3y$

Entón:

$$\begin{cases} 0 \le x \le 3 \\ \frac{3}{2}x < y < \frac{9-x}{3} \\ 0 < z < \frac{18-2x-6y}{9} \end{cases}$$

$$2x+6y+9z-18=0 \Rightarrow z=\frac{18-2x-6y}{9}$$

Polo tanto o volumen:

$$\int_0^2 \int_{\frac{3}{2}y}^{9-3y} \int_0^{\frac{18-2x-6y}{9}} dzdxdy$$

11.

$$S=\left\{(x,y,z) \in \mathbb{R}^3 / x^2+y^2 \le z^2, x^2+y^2+z^2 \le 2, z \le 0\right\}$$

Coordenadas cilíndricas:

$$\begin{cases} x=\rho cos\theta \\ y=\rho sen\theta \\ z=z \end{cases}$$

Hallamos a intersección e temos que:

$x^2+y^2\leq 1 \Rightarrow \rho \in [0,1], \theta \in [0,2\pi]$

$z^2 \leq 2-x^2-y^2 \Rightarrow z \leq \pm(-)\sqrt{2-\rho^2}$

$x^2+y^2 \leq z^2 \Rightarrow z^2 \geq x^2+y^2 \Rightarrow z \geq \pm(-)\sqrt{\rho^2} = \pm(-)\rho$

Polo tanto:

$$V=\int_0^1\int_0^{2\pi}\int_{-\sqrt{2-\rho^2}}^{-\rho} \rho dz d\theta d\rho$$

12.

$x=\rho sen\varphi\cos\theta$
$y=\rho sen\varphi sen\theta$
$z=\rho cos\varphi$

jacobiano $\rho^2 sen\varphi$

$\theta\in[0,2\pi]$
$\varphi\in[0,\pi]$

A esfera ven dada por $\rho=\sqrt{2}$

O cono, en coordenadas esféricas ven dado por $x^2+y^2=z^2$

$\rho^2 sen^2\varphi = \rho^2 cos^2\varphi \Rightarrow tg^2\varphi = 1 \Rightarrow tg\varphi = \pm 1$, o considerar a parte inferior;

$\Rightarrow tg\varphi=-1 \Rightarrow \varphi = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

Polo tanto:

$0\leq\rho\leq\sqrt{2}$
$0<\theta<2\pi$
$\dfrac{3\pi}{4}<\varphi<\pi$

$$V=\int_0^{\sqrt{2}}\int_0^{2\pi}\int_{\frac{3\pi}{4}}^{\pi} \rho^2 sen\varphi d\varphi d\theta d\rho$$

13.

$divf \;\; NE$	$\nabla f \;\; C.V.$	$rotF \;\; C.V.$
$\nabla\nabla f \;\; C.E.$	$\nabla\times\nabla f \;\; C.V.$	$divF \;\; C.E.$

14.

$$\sigma(t) = (sent, \cos t, 4t)$$
$$\sigma'(t) = (\cos t, -sent, 4)$$
$$\sigma'(2\pi) = (1,0,4) \leftarrow velocidad$$
$$rapidez = \|velocidad\| = \sqrt{1^2+0^2+4^2} = \sqrt{17}$$

15.

Parametrizamos a superficie:

$x-1 = \rho\cos\theta$ $\qquad (x-1)^2 + y^2 = (z+2)^2$

$y = \rho sen\theta$ $\qquad (z+2)^2 = \rho^2 \Rightarrow (z+2) = \pm\rho \Rightarrow z = -2\pm\rho$

Como solo me interesa a parte de arriba de arriba de cómo z=-2 + ρ .

Por tanto:

$S(\rho,\theta) = (1+\rho\cos\theta, \rho sen\theta, -2+\rho)$ $\qquad z\in[1,2] \Rightarrow z+2 \in [3,4] \Rightarrow \rho\in[3,4]$

$$S\rho = (\cos\theta, sen\theta, 1)$$
$$S\theta = (-\rho sen\theta, \rho\cos\theta, 0)$$

$$S\rho \; x \; S\theta = \begin{vmatrix} i & j & k \\ \cos\theta & sen\theta & 1 \\ -\rho sen\theta & \rho\cos\theta & 0 \end{vmatrix} = \rho\cos^2\theta\vec{k} - \rho sen\theta\vec{j} - (-\rho sen^2\theta\vec{k} + \rho\cos\theta\vec{i}) = (-\rho\cos\theta, -\rho sen\theta, \rho)$$

$$\|S\theta \, x \, S\rho\| = \sqrt{\rho^2 \cos^2\theta + \rho^2 sen^2\theta + \rho^2} = \sqrt{2\rho^2} = \sqrt{2}\rho$$

$$A = \int_3^4 \int_0^{2\pi} \sqrt{2}\rho.\rho \, d\theta d\rho$$

16.

$$\begin{aligned} x-1 &= 2\cos t \\ y-2 &= 2sent \end{aligned} \quad t \in [-2\pi, 0]$$ Para recorrela o revés

$$\sigma(t) = (1+2\cos t, 2+2sent)$$

- $(1+\sqrt{3}, 3)$

$$\left.\begin{aligned} 1+\sqrt{3} = 1+2\cos t \Rightarrow \cos t = \frac{\sqrt{3}}{2} \\ 3 = 2+2sent \Rightarrow 2sent = 1 \Rightarrow sent = \frac{1}{2} \end{aligned}\right\} \Rightarrow t = -2\pi - \frac{\pi}{6} = -\frac{13\pi}{6}$$

- $(2, \sqrt{3}+2)$

$$\left.\begin{aligned} 2 = 1+2\cos t \Rightarrow 2\cos t = 1 \Rightarrow \cos t = \frac{1}{2} \\ \sqrt{3+2} = 2+2sent \Rightarrow 2sent = \sqrt{3} \Rightarrow sent = \frac{\sqrt{3}}{2} \end{aligned}\right\} \Rightarrow t = -\frac{\pi}{3}$$

Polo tanto,

$$L = \int_{-\frac{13\pi}{6}}^{-\frac{\pi}{3}} 2dt = 2t\Big|_{-\frac{13\pi}{6}}^{-\frac{\pi}{3}} = -\frac{2\pi}{3} - \left(-\frac{26\pi}{6}\right) = \frac{22\pi}{6} = \frac{11\pi}{3}$$

17.

$\boldsymbol{r(t) = (2\cos t, 2sent) \quad t \in [0, \pi]}$

$\boldsymbol{\rho(x,y) = 3y}$

$$m = \int_0^{\pi} \rho(r(t)) \cdot \|r'(t)\| dt = \int_0^{\pi} 12sentdt = 12(-\cos t)_0^{\pi} = 12 - (-12) = 24$$

$\rho\left(r\left(t\right)\right)=6sent$

$r'\left(t\right)=\left(-2sent,2\cos t\right)$

$\left\|r'\left(t\right)\right\|=\sqrt{4sen^2t+4cos^2t}=\sqrt{4}=2$

18.

$F\left(x,y,z\right)=\left(y,-x,0\right) \qquad x^2+y^2=1 \quad x+y+z=1$

A intersección ven dada por:

$$\begin{cases} x^2+y^2=1 \\ z=1-x-y \end{cases}$$

Polo tanto:

$r\left(\theta\right)=\left(\cos\theta,\mathrm{s}en\theta,1-\cos\theta-\mathrm{s}en\theta\right) \;\; \theta\in\left[0,2\pi\right]$

$r\left(0\right)=\left(1,0,0\right)$

$r\left(\frac{\pi}{2}\right)=\left(0,1,0\right)$

$r'\left(\theta\right)=\left(-\mathrm{s}en\theta,\cos\theta,\mathrm{s}en\theta-\cos\theta\right) \;\; \theta\in\left[0,2\pi\right]$

Entón:

$$\int_0^{2\pi}\left(\mathrm{s}en\theta,-\cos\theta,0\right)\cdot\left(-\mathrm{s}en\theta,\cos\theta,\mathrm{s}en\theta-\cos\theta\right)d\theta=\int_0^{2\pi}\left(-\mathrm{s}en^2\theta-\cos^2\theta\right)d\theta=\int_0^{2\pi}-1d\theta=-2\pi$$

$F\left(r\left(\theta\right)\right)=\left(\mathrm{s}en\theta,-\cos\theta,0\right)$

19.

$S\left(x,y\right)=\left(x,y,3-x-y\right)$

$$S_x \times S_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \vec{k} - \left(-\vec{i} - \vec{j}\right) = \vec{i} + \vec{j} + \vec{k} = (1,1,1)$$

$$S_x = (1,0,-1) \quad x \in [0,3]$$
$$S_y = (0,1,-1) \quad y \in [0,3-x]$$

$\iint_{S_1} F dS_1 = -\iint_{S_2} F dS_2$ Cando S_1 e S_2 teñen orientacións distintas.

Polo tanto:

$$\iint_S F dS = -\int_0^3 \int_0^{3-x} (x, y, 3-x-y) \cdot (1,1,1)\, dydx =$$

$$F\left(S(x,y)\right) = (x, y, 3-x-y)$$

$$= -\int_0^3 \int_0^{3-x} \left(\cancel{x} + \cancel{y} + 3 - \cancel{x} - \cancel{y}\right) dydx = -3\int_0^3 \int_0^{3-x} dudx =$$

$$-3\int_0^3 y\Big|_0^{3-x} dx = -3\int_0^3 (3-x)\, dx = -3\left(3x - \frac{x^2}{2}\right)_0^3 =$$

$$= -3\left(9 - \frac{9}{2}\right) = -\left(27 - \frac{27}{2}\right) = -\frac{27}{2}$$