1. Dados os vectores $\mathbf{u}=(1,0,1)$ e $\mathbf{v}=(0,1,1)$ calcula un vector $\mathbf{w}$ tal que $\mathbf{w}\perp\mathbf{u}$, $\mathbf{w}\perp\mathbf{v}$ e $\|\mathbf{w}\|=\sqrt{3}$.

2. As rectas $\mathbf{r}_1 = \mathbf{x} = \frac{\mathbf{y}-1}{2} = \frac{\mathbf{z}-2}{3}$ e $\mathbf{r}_2 = \frac{\mathbf{x}-3}{2} = \frac{\mathbf{y}-2}{4} = \frac{\mathbf{z}-1}{6}$

   a) Córtanse nun só punto b) son paralelas e distintas

   c) Son a mesma recta d) Crúzanse

3. Atopa un intervalo $[0,\mathbf{b}]$ e unha función $\mathbf{g}:[0,\mathbf{b}]\subset\mathbb{R}\to\mathbb{R}^2$ que verifique que $\mathbf{g}([\mathbf{a},\mathbf{b}])$ é a semielipse da ecuación:

$$(\mathbf{x}-1)^2+4\mathbf{y}^2=1 \text{ con } \mathbf{y}\geq 0$$

4. Escribe **E, P** ou **H**

   a) $25\mathbf{x}^2+9\mathbf{y}^2-100\mathbf{x}+54\mathbf{y}-44=0$ b) $\mathbf{y}^2+\mathbf{x}+\mathbf{y}=0$

   c) $4(\mathbf{x}-2)^2-9(\mathbf{y}-3)^2=36$ d) $4\mathbf{y}^2=\mathbf{x}^2-4\mathbf{x}$

5. Escribir N (Se non ten solución); E (se é un escalar) ou V (se é vectorial)

   a) $(\mathbf{u}\ \mathbf{x}\ \mathbf{v})\cdot\mathbf{w}$ b) $\lambda\mathbf{u}\ \mathbf{x}(\mu\mathbf{v}\ \mathbf{x}\ \mathbf{w})$

   c) $\lambda\mathbf{u}\cdot(\mu\mathbf{v}-\mathbf{w})$ d) $\mathbf{u}+(\lambda\mathbf{v}\cdot\mu\mathbf{w})$

6. Describe e fai un esbozo de $6\mathbf{z}=\mathbf{y}^2-\mathbf{x}^2$

7. Dominio de:

a) $\mathbf{f}(\mathbf{x},\mathbf{y})=1+\sqrt[3]{-(\mathbf{x}-\mathbf{y})^2}$

b) $\mathbf{f}(\mathbf{x},\mathbf{y})=\dfrac{\mathbf{log}\left(9-\mathbf{x}^2\right)}{1+\sqrt{1+\mathbf{y}^2}}$

c) $\mathbf{f}(\mathbf{x},\mathbf{y})=\dfrac{1}{\mathbf{x}-1}+\dfrac{1}{\mathbf{y}}$

d) $\mathbf{f}(\mathbf{x},\mathbf{y})=\sqrt{1-\mathbf{xy}}$

8. Escribe o interior e a fronteira do dominio do apartado b) do exercicio anterior.

9. Calcular as ecuacións dos planos tanxentes en forma $(\mathbf{Ax}+\mathbf{By}+\mathbf{Cz}+\mathbf{D}=0)$ á gráfica da función $\mathbf{f}(\mathbf{x},\mathbf{y})=\sqrt{\mathbf{x}^2+\mathbf{y}^2}$ nos puntos $(0,0,0)$ e $\left(\sqrt{2},\sqrt{2},2\right)$

10. Dadas as funcións $\mathbf{f}(\mathbf{x},\mathbf{y})=\mathbf{y}\ \mathbf{sen}\dfrac{1}{\mathbf{xy}}$ e $\mathbf{g}(\mathbf{x},\mathbf{y})=\dfrac{\mathbf{y}^2}{\mathbf{x}^2+2\mathbf{y}^2}$, calcula os seguintes límites:

a) $\lim\limits_{\mathbf{x}\to 0}\left(\lim\limits_{\mathbf{y}\to 0}\mathbf{f}(\mathbf{x},\mathbf{y})\right)=$

b) $\lim\limits_{(\mathbf{x},\mathbf{y})\to(0,0)}\mathbf{f}(\mathbf{x},\mathbf{y})=$

c) $\lim\limits_{\mathbf{y}\to 0}\left(\lim\limits_{\mathbf{x}\to 0}\mathbf{f}(\mathbf{x},\mathbf{y})\right)=$

d) $\lim\limits_{\substack{\mathbf{x}\to 0\\ \mathbf{y}=\mathbf{mx}}}\mathbf{g}(\mathbf{x},\mathbf{y})=$

11. Se estamos situados no punto $(2,0,1)$ do espazo, calcula o vector $\mathbf{v}$ que indica cara ónde hai que moverse para que a función $f(x,y,z)=e^{xy}\cos(yz)$ diminúa rápidamente. Calcula tamén a derivada direccional na dirección do vector $(2,1,2)$.

12. Calcula e clasifica os puntos críticos de $f(x,y)=3x^2-x^3-9y-3y^2+y^3$.

13. Atopa os extremos absolutos de $f(x,y)=3x^2-x^3-9y-3y^2+y^3$ no conxunto $S=\{(x,y)\in\mathbb{R}^2 \,/\, x^2+2y^2=6\}$

SOLUCIONES

Dados os vectores $\mathbf{u}=(1,0,1)$ e $\mathbf{v}=(0,1,1)$ calcula un vector $\mathbf{w}$ tal que $\mathbf{w}\perp\mathbf{u}$, $\mathbf{w}\perp\mathbf{v}$ e $\|\mathbf{w}\|=\sqrt{3}$.

$$\begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = \vec{\mathbf{k}} - \left(\vec{\mathbf{i}}+\vec{\mathbf{j}}\right) = (-1,-1,1)$$

$$\|(-1,-1,1)\| = \sqrt{(-1)^2+(-1)^2+1^2} = \sqrt{3}$$

Polo tanto, $\mathbf{w}=(-1,-1,1)$

<u>Otra forma:</u>

$$\mathbf{w}=(\mathbf{a},\mathbf{b},\mathbf{c})$$

$$u.w=0 \Rightarrow (1,0,1)(a,b,c)=a+c=0 \Rightarrow a=-c$$
$$v.w=0 \Rightarrow (0,1,1)(a,b,c)=b+c=0 \Rightarrow b=-c$$

Entonces,

$$\mathbf{w}=(-\mathbf{c},-\mathbf{c},\mathbf{c})$$

$$|w|=\sqrt{3} \Rightarrow \sqrt{c^2+c^2+c^2}=\sqrt{3c^2}=\sqrt{3} \Rightarrow c=\pm 1$$

Por tanto, w = (-1,-1,1) ou ben w=(1,1,-1).

2. $\mathbf{r}_1 = \mathbf{x} = \dfrac{\mathbf{y}-1}{2} = \dfrac{\mathbf{z}-2}{3}$ e $\mathbf{r}_2 = \dfrac{\mathbf{x}-3}{2} = \dfrac{\mathbf{y}-2}{4} = \dfrac{\mathbf{z}-1}{6}$

$$\vec{v_1}=(1,2,3)$$
$$\vec{v_2}=(2,4,6)$$

Os vectores son proporcionais, por tanto, as rectas son paralelas ou coincidentes.

Tomamos o punto (0,1,2) da recta $\mathbf{r}_1$ e vemos si está na recta $\mathbf{r}_2$:

$$\frac{0-3}{2} \neq \frac{1-2}{4}$$

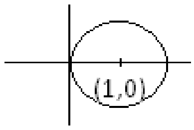
O punto non está na recta, é decir, as rectas son paralelas.

3.

$$(x-1)^2+4y^2=1 \quad y\geq 0$$

$$\left.\begin{array}{r} x-1=2\cos t \\ y=\operatorname{sen} t \end{array}\right\} \Rightarrow \begin{array}{l} x=1+2\cos t \\ y=\operatorname{sen} t \end{array}$$

$$t\in[0,\pi] \; y\geq 0$$



$$g(t)=(1+2\cos t,\operatorname{sen} t) \qquad [a,b]=[0,\pi]$$

4.

a) $25x^2+9y^2-100x+54y-44=0$

Axustando cuadrados temos que:

$$(5x-10)^2+(3y+9)^2-225=0 \Rightarrow \frac{(x-2)^2}{9}+\frac{(y+3)^2}{25}=1$$

Elipse

b)

$$y^2+x+y=0$$

$$\left(y+\frac{1}{2}\right)^2+x-\frac{1}{4}=0 \Rightarrow \left(y+\frac{1}{2}\right)^2=-\left(x-\frac{1}{4}\right)$$

Parábola

c) $4(x-2)^2-9(y-3)^2=36$

$$\frac{(\mathbf{x}-2)^2}{9}-\frac{(\mathbf{y}-3)^2}{4}=1$$

Hipérbola

d) $4\mathbf{y}^2=\mathbf{x}^2-4\mathbf{x}$

$$\frac{(\mathbf{x}-2)^2}{4}-\mathbf{y}^2=1$$

Hipérbola

5.

a) $(\mathbf{u}\ \mathbf{x}\ \mathbf{v})\cdot\mathbf{w}$ Escalar

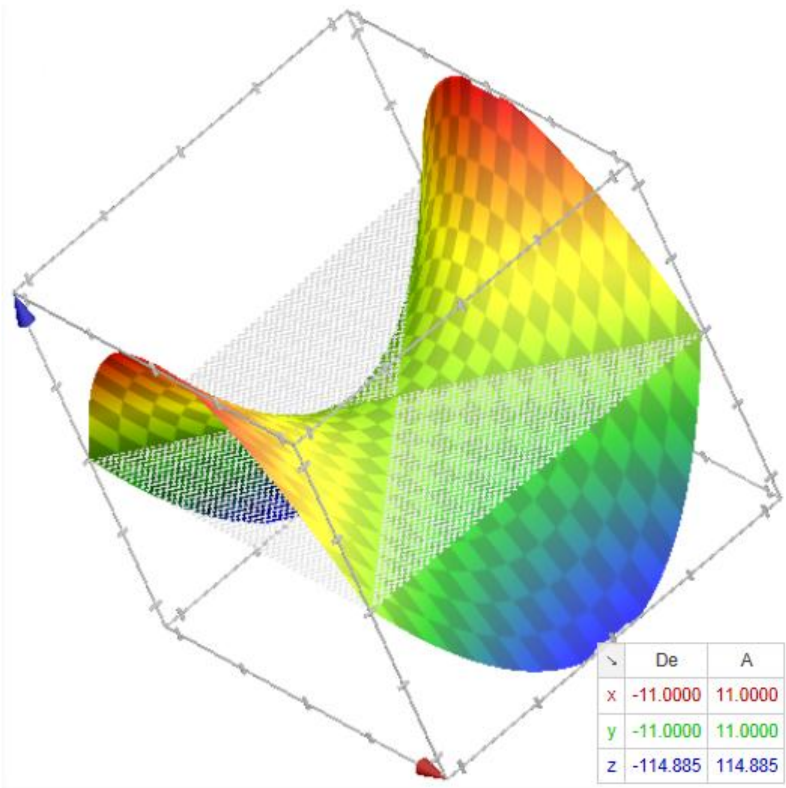
b) $\lambda\mathbf{u}(\mu\mathbf{v}-\mathbf{w})$. Escalar

c) $\lambda\mathbf{u}\times(\mu\mathbf{v}\times\mathbf{w})$. Vector

d) $\mathbf{u}+(\lambda\mathbf{v}\ .\mu\mathbf{w})$ Non ten solución

6.

$6z=y^2-x^2$ É un paraboloide hiperbólico



7.

a) $\mathbf{f}(\mathbf{x},\mathbf{y})=1+\sqrt[3]{-(\mathbf{x}-\mathbf{y})^2}$

Entonces $\mathbf{Dom\ f}=\mathbb{R}^2$

b) $\mathbf{f}(\mathbf{x},\mathbf{y})=\dfrac{\mathbf{log}(9-\mathbf{x}^2)}{1+\sqrt{1+\mathbf{y}^2}}$

$9-x^2>0\Rightarrow(3-x)(3+x)>0\Rightarrow\begin{cases}3-x>0,3+x>0\Rightarrow x<3,x>-3\\3-x<0,3+x<0\Rightarrow x>3,x<-3\end{cases}$

$\mathbf{Dom\ f}=(-3,3)\times\mathbb{R}$

c) $\mathbf{f}(\mathbf{x},\mathbf{y})=\dfrac{1}{\mathbf{x}-1}+\dfrac{1}{\mathbf{y}}$

$\mathbf{x}\neq 1,\mathbf{y}\neq 0$

$\mathbf{Dom\ f}=\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\ /\ \mathbf{x}\neq 1,\mathbf{y}\neq 0\right\}$

d) $\mathbf{f}(\mathbf{x},\mathbf{y})=\sqrt{1-\mathbf{xy}}$

$1-xy\geq 0\Rightarrow xy\leq 1$

$$\mathbf{Dom\ f}=\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\ /\ \mathbf{xy}\leq 1\right\}=\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\ /\ \mathbf{x}=0\right\}\cup$$

$$\cup\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\ /\ \mathbf{x}>0,\ \mathbf{y}\leq\frac{1}{\mathbf{x}}\right\}\cup\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\ /\ \mathbf{x}<0,\mathbf{y}\geq\frac{1}{\mathbf{x}}\right\}$$

8.

$\mathbf{A}=(-3,3)\times\mathbb{R}=\overset{o}{\mathbf{A}}$

$Fr(A)=\left\{(x,y)\in\mathbb{R}^2\ /\ x=-3\right\}\cup\left\{(x,y)\in\mathbb{R}^2\ /\ x=3\right\}$

9.

$$f(x,y)=\sqrt{x^2+y^2} \text{ en } (0,0,0) \text{ e } \left(\sqrt{2},\sqrt{2},2\right)$$

$$\frac{\partial f}{\partial x}(0,0)=\lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{|h|-0}{h}\ \nexists$$

$$\frac{\partial f}{\partial y}(0,0)=\lim_{h\to 0}\frac{f(0,h)-f(0,0)}{h}=\lim_{h\to 0}\frac{|h|-0}{h}\ \nexists$$

Polo tanto, non existe plano tanxente no $(0,0,0)$.

$$\frac{\partial f}{\partial x}=\frac{x}{\sqrt{x^2+y^2}} \qquad \frac{\partial f}{\partial y}=\frac{y}{\sqrt{x^2+y^2}}$$

$$z-2=\frac{\sqrt{2}}{2}\left(x-\sqrt{2}\right)+\frac{\sqrt{2}}{2}\left(y-\sqrt{2}\right)$$

10.

a) $\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0} f(x,y)\right)=\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0} y\,\mathrm{sen}\frac{1}{xy}\right)=\lim\limits_{x\to 0}0=0$

b) $\lim\limits_{(x,y)\to(0,0)} f(x,y)=\lim\limits_{(x,y)\to(0,0)} y\,\mathrm{sen}\left(\frac{1}{xy}\right)=0$

c) $\lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} f(x,y)\right)=\lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} y\,\mathrm{sen}\frac{1}{xy}\right)=\lim\limits_{y\to 0}\left(y\lim\limits_{x\to 0}\mathrm{sen}\left(\frac{1}{xy}\right)\right)\nexists$

d) $\lim\limits_{\substack{x\to 0\\ y=mx}} g(x,y)=\lim\limits_{\substack{x\to 0\\ y=mx}}\frac{m^2x^2}{x^2+2m^2x^2}=\frac{m^2}{1+2m^2}$

11 . $\mathbf{f(x,y,z)=e^{xy}\cos(yz)}$

$$\frac{\partial f}{\partial x}=ye^{xy}\cos(yz)$$
$$\frac{\partial f}{\partial y}=xe^{xy}\cos(yz)-z\cos(yz)e^{xy}$$
$$\frac{\partial f}{\partial z}=-ye^{xy}sen(yz)$$

Por tanto, no punto (2,0,1) temos que:

$\nabla f(2,0,1)=(0,1,0)$ é a dirección na que máis rápidamente aumenta f. Por tanto, na que máis rápidamente diminúe f será na (0,-1,0).

$$D_{\vec{v}}f(2,0,1)=(0,1,0)\left(\tfrac{2}{3},\tfrac{2}{3},\tfrac{1}{3}\right)=\frac{2}{3}$$
$$\vec{v}=(2,2,1)$$
$$\left|\vec{v}\right|=\sqrt{2^2+2^2+1^2}=3$$

12. $\mathbf{f(x,y)=3x^2-x^3-9y-3y^2+y^3}$

$$\frac{\partial f}{\partial x}=6x-3x^2=0\Leftrightarrow\begin{cases}x=0\\x=2\end{cases}$$
$$\frac{\partial f}{\partial y}=-9-6y+3y^2=0\Leftrightarrow\begin{cases}y=3\\y=-1\end{cases}$$

Por tanto, os puntos críticos da función son (0,3), (0,-1),(2,3),(2,-1).

$$\frac{\partial^2 f}{\partial x^2}=6-6x$$
$$\frac{\partial^2 f}{\partial y^2}=-6+6y$$
$$\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}=0$$
$$H(x,y)=\begin{vmatrix}6-6x & 0\\0 & -6+6y\end{vmatrix}$$

Por tanto:

$H(0,3)=\begin{vmatrix}6 & 0\\0 & 12\end{vmatrix}>0$ O punto (0,3) é un punto de mínimo relativo.

$H(0,-1)=\begin{vmatrix}6 & 0\\0 & -12\end{vmatrix}<0$ En (0,-1) hai un punto de sela.

$H(2,3)=\begin{vmatrix}-6 & 0\\0 & 12\end{vmatrix}<0$ En (2,3) hai un punto de sela.

$H(2,-1)=\begin{vmatrix} -6 & 0 \\ 0 & -12 \end{vmatrix} > 0$ En (2,-1) hai un máximo relativo.

13. $f(x,y)=3x^2-x^3-9y-3y^2+y^3$ en $x^2+2y^2=6$

$$\begin{cases} 6x-3x^2=2x\lambda \\ -9-6y+3y^2=4y\lambda \\ x^2+2y^2=6 \end{cases} \Rightarrow \begin{cases} 3x(2-x)=2x\lambda \\ 3(y+1)(y-3)=4y\lambda \\ x^2+2y^2=6 \end{cases}$$

Multiplico a primeira ecuación por2x e a segunda por y, así temos que:

$$6xy(2-x)=4xy\lambda$$

$$3x(y+1)(y-3)=4xy\lambda$$

Por tanto:

$$6xy(2-x)=3x(y+1)(y-3)$$

Así, vemos que a igualdad se cumple para $x=0$ e para os puntos $(2,-1)$ e $(2,3)$. Podría haber máis puntos. Despois resolveremos con wolfram alpha.

Si $x=0 \Rightarrow 2y^2=6 \Rightarrow y=\pm\sqrt{3}$.

Vexamos si os puntos $(2,-1)$ e $(2,3)$ están na elipse:

$2^2+2(-1)^2=6$. Por tanto o punto cumple as tres ecuacións.

$2^2+2.3^2=22\neq 6$. Por tanto, o punto non está na elipse.

Temos así os puntos $(0,\sqrt{3}),(0,-\sqrt{3})$ e $(2,-1)$.

Pode haber mais solucións, hai que resolver o sistema:

$$\begin{cases} 3x(y-3)(y+1)=6yx(2-x) \\ x^2+2y^2=6 \end{cases}$$

Resolvendo o sistema con wolfram alpha obtemos a maiores a solución:

$$x = \frac{2}{9}\left(5 - \frac{26 \times 2^{2/3}}{\sqrt[3]{383 - 81\sqrt{17}}} - \sqrt[3]{2\left(383 - 81\sqrt{17}\right)}\right) \approx -2.41953 \text{ and } y \approx -0.270048$$

Evaluamos a función nos puntos e temos que:

$$f(0, \sqrt{3}) = -19{,}392$$
$$f(0, -\sqrt{3}) = 1{,}3923$$
$$f(2, -1) = 9$$
$$f(-2.41953, -0.270048) = 33.9187$$

Por tanto, a función alcanza un máximo absoluto no punto $\left(-2.41953, -0.27004\right)$ e un mínimo absoluto no punto $(0, \sqrt{3})$.