TIPO C

1. $h(x,y) = \sqrt{\ln(y^2 - x^2)}$

$\ln\left(y^2 - x^2\right) \geq 0 \Rightarrow y^2 - x^2 \geq 1,$ por tanto:

Dom (h) = $\left\{(x,y) \in \mathbb{R}^2 \ / \ y^2 - x^2 \geq 1 \ \right\}$

2. *C* = $\left\{(x,y) \in \mathbb{R}^2 / x^2 > y^2\right\} = \left\{(x,y) \in \mathbb{R}^2 / x^2 - y^2 > 0\right\} = \{(x,y) \in \mathbb{R}^2 / (x-y)(x+y) > 0\} =$

$$(x-y)(x+y) > 0 \Rightarrow \begin{cases} x-y>0, \ x+y>0 \\ ou\ ben \\ x-y<0, x+y<0 \end{cases} \Rightarrow \begin{cases} y<x, y>-x \\ ou\ ben \\ y>x, y<-x \end{cases}$$

$$C = \left\{(x,y) \in \mathbb{R}^2 / y < x, \ \ y > -x\right\} \cup \left\{y > x, \ \ y > -x\right\}$$

Por tanto:

$$Int(C) = C \quad (C \ \ é\ aberto)$$

$$Fr(C) = \left\{(x,y) \in \mathbb{R}^2 / y = x\right\} \cup \left\{(x,y) \in \mathbb{R}^2 / y = -x\right\}$$

3. $f(x,y) = -ye^{\frac{-x^2-y^2}{2}}$

$$\frac{\partial f}{\partial x} = -y.e^{\frac{-x^2-y^2}{2}}.(-x) = xye^{\frac{-x^2-y^2}{2}} = 0 \Leftrightarrow \begin{cases} x=0 \\ y=0 \end{cases}$$

$$\frac{\partial f}{\partial y} = -e^{\frac{-x^2-y^2}{2}} - y. \ e^{\frac{-x^2-y^2}{2}}.(-y) = -e^{\frac{-x^2-y^2}{2}} + y^2e^{\frac{-x^2-y^2}{2}} = 0 \Leftrightarrow y^2 - 1 = 0 \Leftrightarrow y = \pm 1$$

Por tanto, os puntos críticos son:

(0,1) , (0, -1)

4. $f(x,y) = -x^2y \qquad D = \left\{(x,y) \in \mathbb{R}^2 / x^2 + 2y^2 = 6\right\}$

$$\nabla f(x,y) = (-2xy, -x^2)$$
$$\nabla g(x,y) = (2x, 4y)$$

$$\begin{cases} -2xy = 2\lambda x \Leftrightarrow -2x(y+\lambda)=0 \Leftrightarrow \begin{cases} x=0 \\ y=-\lambda \end{cases} \\ -x^2 = 4y\lambda \\ x^2+2y^2=6 \end{cases}$$

- **x=0**

$$x^2+2y^2=6 \Rightarrow 2y^2=6 \Rightarrow y^2=3 \Rightarrow y=\pm\sqrt{3} \quad (0,\sqrt{3})\ ,\ (0,-\sqrt{3})$$

- **y=-** λ

$$-x^2=-4\lambda^2 \Rightarrow x^2=4\lambda^2 \Rightarrow x=\pm 2\lambda$$

$$4\lambda^2+2\lambda^2=6 \Rightarrow \lambda^2=1 \Rightarrow \lambda=\pm 1$$

$$\lambda=1 \Rightarrow y=-1 \Rightarrow \begin{cases} x=2 \\ x=-2 \end{cases} \qquad (2,-1)\ ,\ (-2,-1)$$

$$\lambda=-1 \Rightarrow y=1 \Rightarrow \begin{cases} x=2 \\ x=-2 \end{cases} \qquad (2,1)\ ,\ (-2,1)$$

$$f(0,\sqrt{3})=0$$

$$f(0,-\sqrt{3})=0$$

$$\left.\begin{array}{l} f(2,-1)=4 \\ f(-2,-1)=4 \end{array}\right\} Máximo\ absoluto$$

$$\left.\begin{array}{l} f(2,1)=-4 \\ f(-2,1)=-4 \end{array}\right\} Mínimo\ absoluto$$

5. $x^2-2x+y^2-z^2-4z-3=0$

$$(x-1)^2+y^2-\left(z+2\right)^2=x^2-2x+1+y^2-(z^2+4+4z)=x^2-2x+y^2-z^2-4z-3=0$$

Por tanto:

$$(x-1)^2+y^2-(z+2)^2=0\Rightarrow(z+2)^2=(x-1)^2+y^2$$

Cono de vértice no (1 , 0 , -2)

$x^2-2x+y^2+z^2+4z+5=0$

$$(x-1)^2+y^2+(z+2)^2=x^2-2x+1+y^2+z^2+4z+4=x^2-2x+y^2+z^2+4z+5=0$$

$$(x-1)^2+y^2+(z+2)^2=0\Rightarrow\begin{cases}x-1=0\Rightarrow x=1\\ y=0\\ z+2=0\Rightarrow z=-2\end{cases}$$

É un punto: (1 , 0 , -2)

6. $A=\int_0^2 (-x+1-(-x))dx=\int_0^2 dx = x]_0^2 = 2u^2$

7.

$$\int_0^4 \int_{\frac{x}{2}}^2 f(x,y)dydx$$

$$\begin{cases} 0 < x < 4 \\ \frac{x}{2} < y < 2 \end{cases}$$

$$\begin{cases} 0 < y < 2 \\ 0 < x < 2y \end{cases}$$

$$\int_0^2 \int_0^{2y} f(x,y)dxdy$$

8. $S=\{(x,y,z)\in \mathbb{R}^2 / (x-1)^2+y^2 \le (z+2)^2,\ (x-1)^2+y^2+(z+2)^2 \le 8,\ z\le -2\}$

$$(z+2)^2 \ge (x-1)^2+y^2 \Rightarrow (z+2)\ge \pm\sqrt{(x-1)^2+y^2} \Rightarrow z \ge -2\pm\sqrt{(x-1)^2+y^2}$$

$$(z+2)^2 \le 8-(x-1)^2-y^2 \Rightarrow (z+2)\le \pm\sqrt{8-(x-1)^2-y^2} \Rightarrow z\le -2\pm\sqrt{8-(x-1)^2-y^2} \Rightarrow z\le -2$$

Esfera centrada en (1 , 0 , -2), cono con vértice en (1 , 0 , -2). Hallamos a intersección de esfera e como:

$$\left.\begin{aligned} (z+2)^2 &= (x-1)^2+y^2 \\ (z+2)^2 &= 8-(x-1)^2-y^2 \end{aligned}\right\} \Rightarrow (x-1)^2+y^2 = 8-(x-1)^2-y^2$$

$$2(x-1)^2+2y^2 = 8 \Rightarrow (x-1)^2+y^2 = 4$$

Por tanto:

$-1 \leq x \leq 3$

$-\sqrt{4-(x-1)^2} \leq y \leq \sqrt{4-(x-1)^2}$

$x \in [-1,3]$

$y^2 = 4-(x-1)^2 \Rightarrow y \pm \sqrt{4-(x-1)^2}$

$E\ z,$

$z \geq -2 \pm \sqrt{(x-1)^2 + y^2}$

$z \leq -2 \pm \sqrt{8-(x-1)^2 - y^2}$

Por tanto:

$$V = \int_1^3 \int_{-\sqrt{4-(x-1)^2}}^{\sqrt{4-(x-1)^2}} \int_{-2-\sqrt{8-(x-1)^2-y^2}}^{-2-\sqrt{(x-1)^2+y^2}} dzdydx$$

10. Hallamos o plano que pasa por os puntos (0 , 3 , 0) , (3 , 2 , 0) e (0 , 0 , 3).

(0 , 3 , 0) – (3 , 2 , 0) = (-3 , 1 , 0)

(0, 0 , 3) – (3 , 2 , 0) = (-3 , -2 , 3)

$$\pi : \begin{vmatrix} x & y & z-3 \\ -3 & 1 & 0 \\ -3 & -2 & 3 \end{vmatrix} = 3x + 6(z-3) - (-3(z-3) - 9y) = 3x + 6z - 18 - (-3z + 9 - 9y) =$$

$$= 3x + 6z - 18 + 3z - 9 + 9y \Rightarrow 3x + 9y + 9z - 27 = 0 \Rightarrow \pi : x + 3y + 3z - 9 = 0$$

Calculamos a proyección sobre XY:

Calcula a recta do (0 , 0) ó (3 , 2):

$$\frac{x-3}{3}=\frac{y-2}{2}\Rightarrow 2x$$

$$\frac{x-3}{3}=\frac{y-2}{2}\Rightarrow 2x\cancel{-6}=3y\cancel{-6}\Rightarrow 3y=2x\Rightarrow y=\frac{3}{2}x$$

Polo tanto, a rexión: $0\le x\le 3,\frac{9-x}{3}>y>\frac{3}{2}x$

Entón:

$$\begin{cases} 0\le x\le 3 \\ \frac{3}{2}x<y<\frac{9-x}{3} \\ 0<z<\frac{9-x-3y}{3} \end{cases}$$

$$x+3y+3z-9=0\Rightarrow 3z=9-x-3y\Rightarrow z=\frac{9-x-3y}{3}$$

Polo tanto o volumen:

$$\int_0^3\int_{\frac{3}{2}x}^{\frac{9-x}{3}}\int_0^{\frac{9-x-3y}{3}} dzdydx$$

11.

$$S=\left\{(x,y,z)\in\mathbb{R}^3 / x^2+y^2\le z^2, x^2+y^2+z^2\le 8, z\le 0\right\}$$

Coordenadas cilíndricas:

$$\begin{cases} \boldsymbol{x=\rho cos\theta} \\ \boldsymbol{y=\rho sen\theta} \\ \boldsymbol{z=z} \end{cases}$$

Hallamos a intersección e temos que:

$$x^2+y^2\le 4\Rightarrow \rho\in[0,2],\theta\in[0,2\pi]$$

$$z^2 \le 8 - x^2 - y^2 \Rightarrow z \le \pm(-)\sqrt{8-\rho^2}$$

$$x^2 + y^2 \le z^2 \Rightarrow z^2 \ge x^2 + y^2 \Rightarrow z \ge \pm(-)\sqrt{\rho^2} = \pm(-)\rho$$

Polo tanto:

$$V = \int_0^2 \int_0^{2\pi} \int_{-\sqrt{8-\rho^2}}^{-\rho} \rho dz d\theta d\rho$$

12.

$$x = \rho sen\varphi \cos\theta \qquad y = \rho sen\varphi sen\theta \qquad z = \rho cos\varphi$$

jacobiano $\rho^2 sen\varphi$

$$\theta \in [0, 2\pi] \qquad \varphi \in [0, \pi]$$

A esfera ven dada por $\rho = \sqrt{8}$

O cono, en coordenadas esféricas ven dado por $x^2 + y^2 = z^2$

$\cancel{\rho^2} sen^2\varphi = \cancel{\rho^2} cos^2\varphi \Rightarrow tg^2\varphi = 1 \Rightarrow tg\varphi = \pm 1$, o considerar a parte inferior;

$$\Rightarrow tg\varphi = -1 \Rightarrow \varphi = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Polo tanto:

$$0 \le \rho \le \sqrt{8}$$
$$0 < \theta < 2\pi$$
$$\frac{3\pi}{4} < \varphi < \pi$$

$$V = \int_0^{\sqrt{8}} \int_0^{2\pi} \int_{\frac{3\pi}{4}}^{\pi} \rho^2 sen\varphi d\varphi d\theta d\rho$$

13.

$divf \quad N.E.$	$\nabla f \quad C.V.$	$rotF \quad C.V.$
$\nabla\nabla f \quad C.E.$	$\nabla \times \nabla f \quad C.V.$	$divF \quad C.E.$

14.

$$\sigma(t) = (sent, \cos t, 4t)$$
$$\sigma'(t) = (\cos t, -sent, 4)$$
$$\sigma'(\frac{3\pi}{2}) = (0,1,4) \leftarrow velocidad$$
$$rapidez = \|velocidad\| = \sqrt{0^2 + 1^2 + 4^2} = \sqrt{17}$$

15.

Parametrizamos a superficie:

$x - 1 = \rho\cos\theta \qquad (x-1)^2 + y^2 = (z+2)^2$

$y = \rho sen\theta \qquad (z+2)^2 = \rho^2 \Rightarrow (z+2) = \pm\rho \Rightarrow z = -2 \pm \rho$

Como solo me interesa a parte de arriba de arriba do cono z=-2 + ρ .

Por tanto:

$S(\rho,\theta) = (1 + \rho\cos\theta, \rho sen\theta, -2 + \rho) \qquad z \in [2,3] \Rightarrow z + 2 \in [4,5] \Rightarrow \rho \in [4,5]$

$$S\rho = (\cos\theta, sen\theta, 1)$$
$$S\theta = (-\rho sen\theta, \rho\cos\theta, 0)$$

$$S\rho \; x \; S\theta = \begin{vmatrix} i & j & k \\ \cos\theta & sen\theta & 1 \\ -\rho sen\theta & \rho\cos\theta & 0 \end{vmatrix} = \rho\cos^2\theta\vec{k} - \rho sen\theta\vec{j} - (-\rho sen^2\theta\vec{k} + \rho\cos\theta\vec{i}) = (-\rho\cos\theta, -\rho sen\theta, \rho)$$

$$\|S\theta \, x \, S\rho\| = \sqrt{\rho^2 \cos^2\theta + \rho^2 sen^2\theta + \rho^2} = \sqrt{-2\rho^2} = \sqrt{2}\rho$$

$$A = \int_4^5 \int_0^{2\pi} \sqrt{2}\rho.\rho \, d\theta d\rho$$

16.

$$(x-1)^2 + (y-2)^2 = x^2 - 2x + 1 + y^2 - 4y + 4 \Rightarrow x^2 - 2x + 1 + y^2 - 4y = (x-1)^2 + (y-2)^2 - 4 = 0$$

$\begin{matrix} x-1 = 2\cos t \\ y-2 = 2 sent \end{matrix}$ $\quad t \in [-2\pi, 0]$ Para recorrela o revés

$$\sigma(t) = (1 + 2\cos t, 2 + 2sent)$$

- $(1+\sqrt{3}, 3)$

$$\left.\begin{matrix} 1+\sqrt{3} = 1 + 2\cos t \Rightarrow \cos t = \dfrac{\sqrt{3}}{2} \\ 3 = 2 + 2sent \Rightarrow 2sent = 1 \Rightarrow sent = \dfrac{1}{2} \end{matrix}\right\} \Rightarrow t = -2\pi - \frac{\pi}{6} = -\frac{13\pi}{6}$$

- $(2, \sqrt{3}+2)$

$$\left.\begin{matrix} 2 = 1 + 2\cos t \Rightarrow 2\cos t = 1 \Rightarrow \cos t = \dfrac{1}{2} \\ \sqrt{3+2} = 2 + 2sent \Rightarrow 2sent = \sqrt{3} \Rightarrow sent = \dfrac{\sqrt{3}}{2} \end{matrix}\right\} \Rightarrow t = -\frac{\pi}{3}$$

Polo tanto,

$$L = \int_{-\frac{13\pi}{6}}^{-\frac{\pi}{3}} 2dt = 2t \Big|_{-\frac{13\pi}{6}}^{-\frac{\pi}{3}} = -\frac{2\pi}{3} - \left(-\frac{26\pi}{6}\right) = \frac{22\pi}{6} = \frac{11\pi}{3}$$

17.

$$r(t) = (2cost, 2sent) \qquad t \in [0, \pi]$$

$\rho(x,y)=3y$

$$m=\int_0^{\pi}\rho\left(r(t)\right)\cdot\left\|r'(t)\right\|dt=\int_0^{\pi}12sent dt=12\left(-\cos t\right)_0^{\pi}=12-\left(-12\right)=24$$

$$\rho\left(r(t)\right)=6sent$$

$$r'(t)=\left(-2sent,2\cos t\right)$$

$$\left\|r'(t)\right\|=\sqrt{4sen^2t+4cos^2t}=\sqrt{4}=2$$

18.

$$F(x,y,z)=(y,-x,0)\qquad x^2+y^2=1\quad x+y+z=1$$

A intersección ven dada por:

$$\begin{cases}x^2+y^2=1\\ z=1-x-y\end{cases}$$

Polo tanto:

$$r(\theta)=\left(\cos\theta,sen\theta,1-\cos\theta-sen\theta\right)\ \theta\in[0,2\pi]$$

$$r(0)=(1,0,0)$$

$$r\left(\frac{\pi}{2}\right)=(0,1,0)$$

$$r'(\theta)=\left(-sen\theta,\cos\theta,sen\theta-\cos\theta\right)\ \theta\in[0,2\pi]$$

Entón:

$$\int_0^{2\pi}\left(sen\theta,-\cos\theta,0\right)\cdot\left(-sen\theta,\cos\theta,sen\theta-\cos\theta\right)d\theta=\int_0^{2\pi}\left(-sen^2\theta-\cos^2\theta\right)d\theta=\int_0^{2\pi}-1d\theta=-2\pi$$

$$F\left(r(\theta)\right)=\left(sen\theta,-\cos\theta,0\right)$$

19.

$S(x,y)=(x,y,3-x-y)$

$$S_x\times S_y=\begin{vmatrix} i & j & k\\ 1 & 0 & -1\\ 0 & 1 & -1\end{vmatrix}=\vec{k}-\left(-\vec{i}-\vec{j}\right)=\vec{i}+\vec{j}+\vec{k}=(1,1,1)$$

$$S_x=(1,0,-1) \quad x\in[0,3]$$
$$S_y=(0,1,-1) \quad y\in[0,3-x]$$

$\iint_{S_1} F dS_1=-\iint_{S_2} F dS_2$ Cando S_1 e S_2 teñen orientacións distintas.

Polo tanto:

$$\iint_S F dS=-\int_0^3\int_0^{3-x}(x,y,3-x-y)\cdot(1,1,1)\,dydx=$$

$$F(S(x,y))=(x,y,3-x-y)$$

$$=-\int_0^3\int_0^{3-x}\left(\cancel{x}+\cancel{y}+3-\cancel{x}-\cancel{y}\right)dydx=-3\int_0^3\int_0^{3-x}dudx=$$

$$-3\int_0^3 y\Big|_0^{3-x}dx=-3\int_0^3(3-x)\,dx=-3\left(3x-\frac{x^2}{2}\right)_0^3=$$

$$=-3\left(9-\frac{9}{2}\right)=-\left(27-\frac{27}{2}\right)=-\frac{27}{2}$$