

1.- Sexa $\mathbf{u} \in \mathbb{R}^2$ / $\|\mathbf{u}\| = 2$ e forma un ángulo de $\frac{\pi}{4}$ con $\mathbf{v} = (1,1)$. Indica cal afirmación é correcta:

- a) \mathbf{u} e \mathbf{v} son \perp b) $\|\mathbf{v}\| = 1$ c) $\mathbf{u} \cdot \mathbf{v} = 2$ d) $\|\mathbf{u} + \mathbf{v}\| = 2 + \sqrt{2}$

2.- Escribe **E, P ou H** segundo corresponda:

- a) $9(\mathbf{x}-2)^2 - 4(\mathbf{y}+3)^2 = 36$ **H** b) $\mathbf{x}^2 + 2\mathbf{x} - 4\mathbf{y} - 3 = 0$ **P**
 c) $9(\mathbf{x}-4)^2 + 16(\mathbf{y}-3)^2 = 166$ **E** d) $25\mathbf{x}^2 + 9\mathbf{y}^2 - 100\mathbf{x} + 54\mathbf{y} - 44 = 0$ **E**

3.- Sexa $\lambda, \mu \in \mathbb{R}$ e $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ sinala **N** (\cancel{N}), **E** (escalar) ou **V** (vector).

- a) $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$ b) $\lambda \mathbf{u} (\mu \mathbf{v} \times \mathbf{w})$ c) $\lambda \mathbf{u} \times (\mu \mathbf{v} - \mathbf{w})$ d) $\lambda \mathbf{u} + (\mathbf{v} \times \mu \mathbf{w})$

4.- Describe e fai un esbozo da superficie cuádrica dada por: $16\mathbf{x}^2 - \mathbf{y}^2 + 16\mathbf{z}^2 = 4$

5.- Escribe o dominio:

- a) $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sqrt[3]{\frac{\mathbf{x}+2}{\mathbf{seny}}}$ b) $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \log(\mathbf{x} + \mathbf{y})$
 c) $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \frac{\sqrt{\mathbf{x}^2 + \mathbf{y}^2 - 9}}{\mathbf{x} - 1}$ d) $\mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = e^{\mathbf{x}/\mathbf{y}} + e^{\mathbf{z}/\mathbf{y}}$

6.- Interior y frontera de b)

7.- Calcula as ecuacións dos planos tanxentes á gráfica da función $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sqrt{\mathbf{x}^2 + \mathbf{y}^2}$ en $(0,0,0)$ e $(1,1\sqrt{2})$

$$\pi(0,0,0) \equiv$$

$$\pi \cdot (1,1,\sqrt{2}) \equiv$$

8.- Dado $f(\mathbf{x}, \mathbf{y}) = \mathbf{x} \operatorname{sen} \frac{1}{\mathbf{xy}}$ e $\nabla(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{xy}}{\mathbf{x}^2 + 2\mathbf{y}^2}$ calcula:

$$\lim_{\mathbf{x} \rightarrow 0} \left(\lim_{\mathbf{y} \rightarrow 0} f(\mathbf{x}, \mathbf{y}) \right) =$$

$$\lim_{\mathbf{y} \rightarrow 0} \left(\lim_{\mathbf{x} \rightarrow 0} f(\mathbf{x}, \mathbf{y}) \right) =$$

$$\lim_{(\mathbf{x}, \mathbf{y}) \rightarrow (0,0)} f(\mathbf{x}, \mathbf{y}) =$$

$$\lim_{\mathbf{x} \rightarrow 0} g(\mathbf{x}, \mathbf{y}) =$$

$$\mathbf{y} = m\mathbf{x}$$

9.- Estamos en $(1,0,1)$, sabiendo esto, calcula el vector \mathbf{v} que indica cara onde hai que moverse para que $f(\mathbf{x}, \mathbf{y}, \mathbf{z}) = e^{\mathbf{xy}} \cos(\mathbf{yz})$, crezca los más rápidamente posible. Calcula también la derivada direccional no sentido do vector $(1,2,2)$.

10.- Calcular os puntos críticos da función $f(\mathbf{x}, \mathbf{y}) = 9\mathbf{x} + 3\mathbf{x}^2 - \mathbf{x}^3 - 3\mathbf{y}^2 + \mathbf{y}^3$.

11.- Atopa os extremos absolutos da función $f(\mathbf{x}, \mathbf{y}) = \mathbf{x}^2 \cdot \mathbf{y}$ no conxunto

$$\mathbf{g} = \left\{ (\mathbf{x}, \mathbf{y}) \in \mathbb{R}^2 \mid \mathbf{x}^2 + 2\mathbf{y}^2 = 6 \right\}.$$

SOLUCION

1.

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos \angle(\vec{u}, \vec{v})$$

$$\vec{u} \cdot \vec{v} = \cancel{2} \cdot \sqrt{2} \frac{\sqrt{2}}{\cancel{2}} = 2$$

$\vec{u} \cdot \vec{v} = 2$, polo tanto, a resposta correcta é a c).

2.

a) $9(x-2)^2 - 4(y+3)^2 = 36$. Dividimos por 36:

$$\frac{(x-2)^2}{4} - \frac{(y+3)^2}{9} = 1 \Rightarrow \left(\frac{x-2}{4}\right)^2 - \left(\frac{y-(-3)}{3}\right)^2 = 1$$

b) $x^2 + 2x - 4y - 3 = 0 \Rightarrow 4y = x^2 + 2x - 3$ ← sumamos y restamos 1

$$4y = x^2 + 2x + 1 - 4 \Rightarrow 4y + 4 = x^2 + 2x + 1 \Rightarrow$$

$$\Rightarrow 4(y+1) = (x+1)^2 \Rightarrow \boxed{4(y-(-1)) = (x-(-1))^2}$$

c) $9(x-4)^2 + 16(y-3)^2 = 166$. Dividimos por 166:

$$\frac{9(x-4)^2}{166} + \frac{8(y-3)^2}{83} = 1 \Rightarrow \frac{(x-4)^2}{\frac{166}{9}} + \frac{(y-3)^2}{\frac{83}{8}} = 1 \Rightarrow$$

$$\boxed{\frac{(x-4)^2}{\left(\frac{\sqrt{166}}{3}\right)^2} + \frac{(y-3)^2}{\left(\frac{\sqrt{83}}{2\sqrt{2}}\right)^2} = 1}$$

$$d) 25x^2 + 9y^2 - 100x + 54y - 44 = 0$$

$$(5x-10)^2 = 25x^2 - 100x + 100$$

$$(3y+9)^2 = 9y^2 + 54y + 81$$

Polo tanto,

$$25x^2 + 9y^2 - 100x + 54y - 44 = (5x-10)^2 + (3y+9)^2 - 225 = 0 \Rightarrow$$

$$\Rightarrow (5(x-2))^2 + (3(y+3))^2 = 225 \Rightarrow$$

$$\Rightarrow 25(x-2)^2 - 9(y-3)^2 = 225 \Rightarrow \frac{(x-2)^2}{9} + \frac{(y-(-3))^2}{25} = 1$$

$$\Rightarrow \frac{(x-2)^2}{3^2} + \frac{(y-(-3))^2}{5^2} = 1$$

3.

$$a) (\vec{U} \cdot \vec{V}) \times \vec{W} \quad \mathbf{N}$$

$\vec{U} \cdot \vec{V}$ é un número, polo tanto, non se pode facer o produto vectorial con \vec{W} .

$$b) \lambda \vec{u} \cdot (\mu \vec{v} \times \vec{w}) \quad \mathbf{V}$$

$\mu \vec{v} \times \vec{w}$ é un vector e si se fai o produto escalar de ese vector co vector $\lambda \vec{u}$ da un escalar.

$$c) \lambda \vec{u} \cdot (\mu \vec{v} - \vec{w}) \quad \mathbf{V}$$

$\mu \vec{v} - \vec{w}$ é un vector e si se fai o produto vectorial con $\lambda \vec{u}$ da outro vector.

d) $\lambda \vec{u} \cdot \underbrace{(\vec{v} \times \mu \vec{w})}_{\text{vector}} \quad \mathbf{V}$ A suma de vectores é un vector.

4.

$$16x^2 - y^2 + 16z^2 = 4 \Rightarrow 4x^2 - \frac{y^2}{4} + 4z^2 = 1 \Rightarrow$$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2} - \frac{y^2}{2^2} + \frac{z^2}{\left(\frac{1}{2}\right)^2} = 1 \quad \text{Hiperboloide de unha folla.}$$

5.

a) $f(x, y) = \sqrt[3]{\frac{x+2}{\text{sen } y}}$

$$f(x, y) = \sqrt[3]{\frac{x+2}{\text{sen } y}} \text{ sen } y \neq 0 \Leftrightarrow y = k\pi, k \in \mathbb{Z}$$

Polo tanto, $\text{Dom}(f) = \{(x, y) \in \mathbb{R}^2 / y \neq k\pi, k \in \mathbb{Z}\}$

b) $f(x, y) = \log(x + y)$

$$x + y > 0 \Rightarrow y > -x$$

$$\text{Dom}(f) = \{(x, y) \in \mathbb{R}^2 / y > -x\}$$

c) $f(x, y) = \frac{\sqrt{x^2 + y^2 - 9}}{x - 1}$

$$x^2 + y^2 - 9 \geq 0 \Leftrightarrow x^2 + y^2 \geq 9 \quad x - 1 = 0 \Leftrightarrow x = 1$$

$$\text{Dom}(f) = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 \geq 9, x \neq 1\}$$

$$d) f(x, y, z) = e^{x/y} + e^{z/y}$$

$$\text{Dom}(f) = \{(x, y, z) \in \mathbb{R}^3 / y \neq 0\}$$

6. Interior y frontera de b)

$$\text{Dom}(f) = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 \geq 9, x \neq 1\}$$

$$\text{Int}(B) = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 > 9, x \neq 1\}$$

$$\text{Fr}(B) = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 = 9\} \cup \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 > 9, x = 1\}$$

7.

$$f(x, y) = \sqrt{x^2 + y^2} \text{ en } (0, 0, 0)$$

$$\frac{\partial f}{\partial x}(0, 0) = \lim_{h \rightarrow 0} \frac{f(0, h, 0) - f(0, 0)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{h^2}}{h} = \lim_{h \rightarrow 0} \frac{|h|}{h} \quad \nexists$$

$$\frac{\partial f}{\partial y}(0, 0) = \lim_{h \rightarrow 0} \frac{f(0, h) - f(0, 0)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{h^2}}{h} = \lim_{h \rightarrow 0} \frac{|h|}{h} \quad \nexists$$

A función non é diferenciable en $(0, 0)$, polo tanto non admite plano tangente en $(0, 0)$.

En $(1, 1, \sqrt{2})$:

$$\frac{\partial f}{\partial x} = \frac{2x}{2\sqrt{x^2 + y^2}} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial y} = \frac{2y}{2\sqrt{x^2 + y^2}} = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x}(1,1) = \frac{1}{\sqrt{2}} \quad \frac{\partial f}{\partial y}(1,1) = \frac{1}{\sqrt{2}}$$

Polo tanto, o plano tangente:

$$z - \sqrt{2} = \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$z - \sqrt{2} = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}} \Rightarrow \boxed{z = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y}$$

8.

$$\lim_{x \rightarrow 0} \left(\lim_{y \rightarrow 0} f(x, y) \right) = \lim_{x \rightarrow 0} \left(\lim_{y \rightarrow 0} x \cdot \frac{1}{xy} \right) =$$

$$= \lim_{x \rightarrow 0} \left(\lim_{y \rightarrow 0} \frac{x \cdot \frac{1}{xy} \cdot \frac{1}{xy}}{\frac{1}{xy}} \right) = \lim_{x \rightarrow 0} \left(\lim_{y \rightarrow 0} \frac{1}{y} \cdot \frac{\frac{1}{xy}}{\frac{1}{xy}} \right) \neq$$

$$\left\{ \begin{array}{l} \lim_{y \rightarrow 0^-} \frac{1}{y} \cdot \frac{\frac{1}{xy}}{\frac{1}{xy}} = -\infty \\ \lim_{y \rightarrow 0^+} \frac{1}{y} \cdot \frac{\frac{1}{xy}}{\frac{1}{xy}} = +\infty \end{array} \right.$$

$$\lim_{y \rightarrow 0} \left(\lim_{x \rightarrow 0} f(x, y) \right) = \lim_{y \rightarrow 0} \left(\lim_{x \rightarrow 0} x \cdot \frac{1}{xy} \right) =$$

$$\lim_{y \rightarrow 0} \left(\lim_{x \rightarrow 0} \frac{1}{y} \cdot \frac{\frac{1}{xy}}{\frac{1}{xy}} \right) = \lim_{y \rightarrow 0} \frac{1}{y} \neq$$

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) = \lim_{\rho \rightarrow 0} \rho \cos \theta \cdot \text{sen} \left(\frac{1}{\rho^2 \cos \theta \cdot \text{sen} \theta} \right) =$$

$$= \lim_{\rho \rightarrow 0} \rho \cos \theta \cdot \frac{1}{\rho^2 \cos \theta \cdot \text{sen} \theta} = \lim_{\rho \rightarrow 0} \frac{1}{\rho \text{sen} \theta} \neq$$

$$= \lim_{\rho \rightarrow 0} \frac{1}{\rho \cdot \text{sen} \theta} = \pm \infty. \text{ Por tanto, o límite non existe.}$$

$$\lim_{x \rightarrow 0} g(x,y) = \lim_{x \rightarrow 0} \frac{x \cdot mx}{x^2 + 2(mx)^2} = \lim_{x \rightarrow 0} \frac{mx^2}{x^2 + 2m^2 x^2} =$$

$$y = mx$$

$$= \lim_{x \rightarrow 0} \frac{m}{1 + 2m^2} = \frac{m}{1 + 2m^2}$$

9.

$$f(x,y,z) = e^{xy} \cdot \cos(yz)$$

$$\frac{\partial f}{\partial x} = e^{xy} \cdot y \cdot \cos(yz)$$

$$\frac{\partial f}{\partial x} = e^{xy} \cdot y \cdot \cos(yz) - e^{xy} \cdot \text{sen}(yz) \cdot z$$

$$\frac{\partial f}{\partial x} = -e^{xy} \cdot \text{sen}(yz) \cdot y$$

$$\frac{\partial f}{\partial x}(1,0,1) = 0 \qquad \frac{\partial f}{\partial y}(1,0,1) = \cos 0 = 1$$

$$\frac{\partial f}{\partial z}(1,0,1) = 0$$

A dirección na que hai que moverse é na dirección $(0,1,0)$, e dicir, na dirección do semieje

OY positiva.

Derivada direccional no sentido $(1, 2, 2)$:

$$D_{\vec{v}}f(1,0,1) = \nabla f(1,0,1) \cdot \vec{v}$$

$$\vec{w} = (1, 2, 2) \Rightarrow \|\vec{w}\| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$\vec{v} = \frac{1}{3}(1, 2, 2)$$

$$D_{\vec{v}}f(1,0,1) = (0, 1, 0) \cdot \frac{1}{3}(1, 2, 2) = \frac{2}{3}$$

10.

$$f(x, y) = 9x + 3x^2 - x^3 - 3y^2 + y^3$$

$$\nabla f(x, y) = (9 + 6x - 3x^2, -6y + 3y^2) = (0, 0) \Rightarrow$$

$$\Rightarrow 9 + 6x - 3x^2 = 0 \Leftrightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow -6y + 3y^2 = 0 \Rightarrow 3y(y - 2) = 0 \begin{cases} \nearrow y = 0 \\ \searrow y = 2 \end{cases}$$

$$x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} \nearrow x = 3 \\ \searrow x = -1 \end{cases}$$

Por tanto, os puntos críticos da función son:

$$(3, 0), (3, 2), (-1, 0), (-1, 2)$$

11.

$$f(\mathbf{x}, \mathbf{y}) = \mathbf{x}^2 \mathbf{y} \qquad \mathbf{f}(\mathbf{x}, \mathbf{y}) = \mathbf{x}^2 \mathbf{y} \qquad \mathbf{B} = \{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^2 / \mathbf{x}^2 + \mathbf{y}^2 = 6\}$$

$$\nabla f(\mathbf{x}, \mathbf{y}) = (2\mathbf{x}\mathbf{y}, \mathbf{x}^2) \qquad \nabla g(\mathbf{x}, \mathbf{y}) = (2\mathbf{x}, 4\mathbf{y})$$

$$\left\{ \begin{array}{l} 2\mathbf{x}\mathbf{y} = 2\lambda\mathbf{x} \Rightarrow 2\mathbf{x}(\mathbf{y} - \lambda) = 0 \begin{array}{l} \nearrow \mathbf{x} = 0 \\ \searrow \mathbf{y} = \lambda \end{array} \\ \mathbf{x}^2 = 4\lambda\mathbf{y} \\ \mathbf{x}^2 + 2\mathbf{y}^2 = 6 \end{array} \right.$$

Si $\mathbf{x} = 0$, entonces:

$$4\lambda\mathbf{y} = 0$$

$$2\mathbf{y}^2 = 6 \Rightarrow \mathbf{y}^2 = 3 \Rightarrow \mathbf{y} = \pm\sqrt{3}$$

Polo tanto, $\boxed{(0, \sqrt{3}), (0, -\sqrt{3})}$

Si $\mathbf{y} = \lambda$, entonces:

$$\mathbf{x}^2 = 4\lambda^2 \Rightarrow \mathbf{x} = \pm\sqrt{4\lambda^2} \Rightarrow \mathbf{x} = \pm 2|\lambda|$$

$$\mathbf{x}^2 + \mathbf{y}^2 = 6 \Rightarrow 4\lambda^2 + 2\lambda^2 = 6 \Rightarrow 6\lambda^2 = 6\mathbf{x} \Rightarrow \boxed{\lambda = \pm 1}$$

si $\lambda = 1, \mathbf{y} = 1, \mathbf{x} = 2, \mathbf{x} = -2$ $(2, 1), (-2, 1)$

si $\lambda = -1, \mathbf{y} = -1, \mathbf{x} = 2, \mathbf{x} = -2$ $(2, -1), (-2, -1)$

Evaluamos ahora a función nos puntos:

$$f(0, \sqrt{3}) = f(0, -\sqrt{3}) = 0$$

$$f(2, 1) = f(-2, 1) = 4$$

$$f(2, -1) = f(-2, -1) = -4$$

Por tanto, 4 é o máximo absoluto de f en g e -4 é o mínimo absoluto de f en g .

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