1.- Sexa $\mathbf{u} \in \mathbb{R}^2 \,/\, \|\mathbf{u}\| = 2$ e forma un ángulo de $\frac{\pi}{4}$ con $\mathbf{v} = (1,1)$. Indica cal afirmación é correcta:

a) $\mathbf{u}$ e $\mathbf{v}$ son $\perp$ b) $\|\mathbf{v}\| = 1$ c) $\mathbf{u} \cdot \mathbf{v} = 2$ d) $\|\mathbf{u} + \mathbf{v}\| = 2 + \sqrt{2}$

2.- Escribe **E,P ou H** segundo corresponda:

a) $9(\mathbf{x}-2)^2 - 4(\mathbf{y}+3)^2 = 36$ **H** b) $\mathbf{x}^2 + 2\mathbf{x} - 4\mathbf{y} - 3 = 0$ **P**

c) $9(\mathbf{x}-4)^2 + 16(\mathbf{y}-3)^2 = 166$ **E** d) $25\mathbf{x}^2 + 9\mathbf{y}^2 - 100\mathbf{x} + 54\mathbf{y} - 44 = 0$**E**

3.- Sexa $\lambda, \mu \in \mathbb{R}$ e $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ sinala $\mathbf{N}(\nexists)$, $\mathbf{E}(\mathbf{escalar})$ ou $\mathbf{V}(\mathbf{vector})$.

a) $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$ b) $\lambda \mathbf{u}(\mu \mathbf{v} \times \mathbf{w})$ c) $\lambda \mathbf{u} \times (\mu \mathbf{v} - \mathbf{w})$ d) $\lambda \mathbf{u} + (\mathbf{v} \times \mu \mathbf{w})$

4.- Describe e fai un esbozo da superficie cuádrica dada por: $16\mathbf{x}^2 - \mathbf{y}^2 + 16\mathbf{z}^2 = 4$

5.- Escribe o dominio:

a) $\mathbf{f}(\mathbf{x},\mathbf{y}) = \sqrt[3]{\dfrac{\mathbf{x}+2}{\mathbf{sen y}}}$ b) $\mathbf{f}(\mathbf{x},\mathbf{y}) = \mathbf{log}(\mathbf{x}+\mathbf{y})$

c) $\mathbf{f}(\mathbf{x},\mathbf{y}) = \dfrac{\sqrt{\mathbf{x}^2 + \mathbf{y}^2 - 9}}{\mathbf{x} - 1}$ d) $\mathbf{f}(\mathbf{x},\mathbf{y},\mathbf{z}) = \mathbf{e}^{\mathbf{x}/\mathbf{y}} + \mathbf{e}^{\mathbf{z}/\mathbf{y}}$

6.- Interior y frontera de b)

7.- Calcula as ecuacións dos planos tanxentes á gráfica da función $\mathbf{f}(\mathbf{x},\mathbf{y}) = \sqrt{\mathbf{x}^2 + \mathbf{y}^2}$ en $(0,0,0)$ e $(1,1\sqrt{2})$

$\pi(0,0,0)\equiv$ $\qquad\qquad$ $\pi\cdot(1,1,\sqrt{2})\equiv$

8.- Dado $f(x,y)=x\,\mathrm{sen}\frac{1}{xy}$ e $\nabla(x,y)=\frac{xy}{x^2+2y^2}$ calcula:

$\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0} f(x,y)\right)=$ $\qquad\qquad$ $\lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} f(x,y)\right)=$

$\lim\limits_{(x,y)\to(0,0)} f(x,y)=$ $\qquad\qquad$ $\lim\limits_{\substack{x\to 0\\ y=mx}} g(x,y)=$

9.- Estamos en $(1,0,1)$, sabiendo esto, calcula el vector $\mathbf{v}$ que indica cara onde hai que moverse para que $f(x,y,z)=e^{xy}\cos(yz)$, crezca los más rápidamente posible. Calcula también la derivada direccional no sentido do vector $(1,2,2)$.

10.- Calcular os puntos críticos da función $f(x,y)=9x+3x^2-x^3-3y^2+y^3$.

11.- Atopa os extremos absolutos da función $f(x,y)=x^2\cdot y$ no conxunto $g=\left\{(x,y)\in\mathbb{R}^2 \,\middle|\, x^2+2y^2=6\right\}$.

SOLUCION

1.

$$\vec{U}\cdot\vec{V}=\left\|\vec{U}\right\|\cdot\left\|\vec{V}\right\|\cdot\cos\measuredangle\left(\vec{U},\vec{V}\right)$$

$$\vec{U}\cdot\vec{V}=\cancel{2}\cdot\sqrt{2}\frac{\sqrt{2}}{\cancel{2}}=2$$

$\vec{U}\cdot\vec{V}=2$, polo tanto, a resposta correcta é a c).

2.

a) $9(x-2)^2-4(y+3)^2=36$. Dividimos por 36:

$$\frac{(x-2)^2}{4}-\frac{(y+3)^2}{9}=1\Rightarrow\left(\frac{x-2}{4}\right)^2-\left(\frac{y-(-3)}{3}\right)^2=1$$

b) $x^2+2x-4y-3=0\Rightarrow 4y=x^2+2x\boxed{-3}\leftarrow$ **sumamos y restamos** 1

$$4y=x^2+2x+1-4\Rightarrow 4y+4=x^2+2x+1\Rightarrow$$

$$\Rightarrow 4(y+1)=(x+1)^2\Rightarrow\boxed{4(y-(-1))=(x-(-1))^2}$$

c) $9(x-4)^2+16(y-3)^2=166$. Dividimos por 166:

$$\frac{9(x-4)^2}{166}+\frac{8(y-3)^2}{83}=1\Rightarrow\frac{(x-4)^2}{\frac{166}{9}}+\frac{(y-3)^2}{\frac{83}{8}}=1\Rightarrow$$

$$\boxed{\frac{(x-4)^2}{\left(\frac{\sqrt{166}}{3}\right)^2}+\frac{(y-3)^2}{\left(\frac{\sqrt{83}}{2\sqrt{2}}\right)^2}=1}$$

d) $25\mathbf{x}^2+9\mathbf{y}^2-100\mathbf{x}+54\mathbf{y}-44=0$

$$(5\mathbf{x}-10)^2=25\mathbf{x}^2-100\mathbf{x}+100$$

$$(3\mathbf{y}+9)^2=9\mathbf{y}^2+54\mathbf{y}+81$$

Polo tanto,

$$25\mathbf{x}^2+9\mathbf{y}^2-100\mathbf{x}+54\mathbf{y}-44=(5\mathbf{x}-10)^2+(3\mathbf{y}+9)^2-225=0\Rightarrow$$

$$\Rightarrow(5(\mathbf{x}-2))^2+(3(\mathbf{y}+3))^2=225\Rightarrow$$

$$\Rightarrow 25(\mathbf{x}-2)^2-9(\mathbf{y}-3)^2=255\Rightarrow\frac{(\mathbf{x}-2)^2}{9}+\frac{(\mathbf{y}-(-3))^2}{25}=1$$

$$\Rightarrow\frac{(\mathbf{x}-2)^2}{3^2}+\frac{(\mathbf{y}-(-3))^2}{5^2}=1$$

3.

a) $\left(\vec{\mathbf{U}}\cdot\vec{\mathbf{V}}\right)\times\vec{\mathbf{W}}$ **N**

$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}}$ é un número, polo tanto, non se pode facer o producto vectorial con $\vec{\mathbf{W}}$.

b) $\lambda\vec{\mathbf{u}}\cdot\left(\mu\vec{\mathbf{v}}\times\vec{\mathbf{w}}\right)$ **V**

$\mu\vec{\mathbf{v}}\mathbf{x}\vec{\mathbf{w}}$ é un vector e si se fai o producto escalar de ese vector co vector $\lambda\vec{\mathbf{u}}$ da un escalar.

c) $\lambda\vec{\mathbf{u}}\cdot\left(\mu\vec{\mathbf{v}}-\vec{\mathbf{w}}\right)$ **V**

$\mu\vec{\mathbf{v}}-\vec{\mathbf{w}}$ é un vector e si se fai o producto vectorial con $\lambda\vec{\mathbf{u}}$ da outro vector.

d) $\lambda\vec{u}\cdot\underbrace{\left(\vec{v}\times\mu\vec{w}\right)}_{\text{vector}}$ **V** A suma de vectores é un vector.

4.

$$16x^2-y^2+16z^2=4\Rightarrow 4x^2-\frac{y^2}{4}+4z^2=1\Rightarrow$$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2}-\frac{y^2}{2^2}+\frac{z^2}{\left(\frac{1}{2}\right)^2}=1$$ Hiperboloide de unha folla.

5.

a) $f(x,y)=\sqrt[3]{\frac{x+2}{seny}}$

$f(x,y)=\sqrt[3]{\frac{x+2}{seny}}\quad seny\neq 0\Leftrightarrow y=k\pi, k\in\mathbb{Z}$

Polo tanto, $Dom(f)=\{(x,y)\in\mathbb{R}^2 / y\neq k\pi, k\in\mathbb{Z}\}$

b) $f(x,y)=\log(x+y)$

$x+y>0\Rightarrow y>-x$

$Dom(f)=\{(x,y)\in\mathbb{R}^2 / y>-x\}$

c) $f(x,y)=\frac{\sqrt{x^2+y^2-9}}{x-1}$

$x^2+y^2-9\geq 0\Leftrightarrow x^2+y^2\geq 9$ $\qquad x-1=0\Leftrightarrow x=1$

$Dom(f)=\{(x,y)\in\mathbb{R}^2 / x^2+y^2\geq 9, x\neq 1\}$

d) $f(x,y,z)=e^{x/y}+e^{z/y}$

$$\text{Dom}(f)=\{(x,y,z)\in\mathbb{R}^2 / y\neq 0\}$$

6. Interior y frontera de b)

$$\text{Dom}(f)=\{(x,y)\in\mathbb{R}^2 / x^2+y^2\geq 9, x\neq 1\}$$

$$\text{Int}(B)=\{(x,y)\in\mathbb{R}^2 / x^2+y^2> 9, x\neq 1\}$$

$$\text{Fr}(B)=\{(x,y)\in\mathbb{R}^2 / x^2+y^2=9\}\cup\{(x,y)\in\mathbb{R}^2 / x^2+y^2>9, x=1\}$$

7.

$f(x,y)=\sqrt{x^2+y^2}$ en $(0,0,0)$

$$\frac{\partial f}{\partial x}(0,0)=\lim_{h\to 0}\frac{f(0,h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{\sqrt{h^2}}{h}=\lim_{h\to 0}\frac{|h|}{h}\quad\nexists$$

$$\frac{\partial f}{\partial y}(0,0)=\lim_{h\to 0}\frac{f(0,h)-f(0,0)}{h}=\lim_{h\to 0}\frac{\sqrt{h^2}}{h}=\lim_{h\to 0}\frac{|h|}{h}\quad\nexists$$

A función non é diferenciable en $(0,0)$, polo tanto non admite plano tangente en $(0,0)$.

En $(1,1,\sqrt{2})$:

$$\frac{\partial f}{\partial x}=\frac{2x}{2\sqrt{x^2+y^2}}=\frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial y}=\frac{2y}{2\sqrt{x^2+y^2}}=\frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial x}(1,1)=\frac{1}{\sqrt{2}} \qquad \frac{\partial f}{\partial y}(1,1)=\frac{1}{\sqrt{2}}$$

Polo tanto, o plano tangente:

$$z-\sqrt{2}=\frac{1}{\sqrt{2}}(x-1)+\frac{1}{\sqrt{2}}(y-1)$$

$$z-\sqrt{2}=\frac{1}{\sqrt{2}}x+\frac{1}{\sqrt{2}}y-\frac{2}{\sqrt{2}} \Rightarrow \boxed{z=\frac{1}{\sqrt{2}}x+\frac{1}{\sqrt{2}}y}$$

8.

$$\lim_{x\to 0}\left(\lim_{y\to 0} f(x,y)\right)=\lim_{x\to 0}\left(\lim_{y\to 0} x\,\text{sen}\frac{1}{xy}\right)=$$

$$=\lim_{x\to 0}\left(\lim_{y\to 0}\frac{\cancel{x}\,\text{sen}\,{}^{1}\!/_{xy}}{{}^{1}\!/_{xy}}\cdot\frac{1}{\cancel{x}y}\right)=\lim_{x\to 0}\underbrace{\left(\lim_{y\to 0}\frac{1}{y}\frac{\text{sen}\,{}^{1}\!/_{xy}}{{}^{1}\!/_{xy}}\right)}_{\nexists}$$

$$\begin{cases}\lim_{y\to 0^-}\frac{1}{y}\cdot\frac{\text{sen}\,{}^{1}\!/_{xy}}{{}^{1}\!/_{xy}}=-\infty\\ \lim_{y\to 0^+}\frac{1}{y}\cdot\frac{\text{sen}\,{}^{1}\!/_{xy}}{{}^{1}\!/_{xy}}=+\infty\end{cases}$$

$$\lim_{y\to 0}\left(\lim_{x\to 0} f(x,y)\right)=\lim_{y\to 0}\left(\lim_{x\to 0} x\cdot\text{sen}\frac{1}{xy}\right)=$$

$$\lim_{y\to 0}\underbrace{\left(\lim_{x\to 0} f\frac{1}{y}\cdot\frac{\text{sen}\left({}^{1}\!/_{xy}\right)}{{}^{1}\!/_{xy}}\right)}_{={}^{1}\!/_{y}}=\boxed{\lim_{y\to 0}\frac{1}{y}} \quad \nexists$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{\rho\to 0} \rho\cos\theta \cdot \operatorname{sen}\left(\frac{1}{\rho^2 \cos\theta \cdot \operatorname{sen}\theta}\right) =$$

$$= \lim_{\rho\to 0} \rho\cos\theta \cdot \frac{1}{\rho^2 \cos\theta \cdot \operatorname{sen}\theta} = \lim_{\rho\to 0} \frac{1}{\rho \operatorname{sen}\theta} \quad \nexists$$

$= \lim_{\rho\to 0} \dfrac{1}{\rho \cdot \operatorname{sen}\theta} = \pm\infty$. Por tanto, o límite non existe.

$$\lim_{x\to 0} g(x,y) = \lim_{x\to 0} \frac{x \cdot mx}{x^2 + 2(mx)^2} = \lim_{x\to 0} \frac{m \cancel{x^2}}{\cancel{x^2} + 2m^2 \cancel{x^2}} =$$

$$y = mx$$

$$= \lim_{x\to 0} \frac{m}{1+2m^2} = \frac{m}{1+2m^2}$$

9.

$$f(x,y,z) = e^{xy} \cdot \cos(yz)$$

$$\frac{\partial f}{\partial x} = e^{xy} \cdot y \cdot \cos(yz)$$

$$\frac{\partial f}{\partial x} = e^{xy} \cdot y \cdot \cos(yz) - e^{xy} \cdot \operatorname{sen}(yz) \cdot z$$

$$\frac{\partial f}{\partial x} = -e^{xy} \cdot \operatorname{sen}(yz) \cdot y$$

$$\frac{\partial f}{\partial x}(1,0,1) = 0 \qquad \frac{\partial f}{\partial y}(1,0,1) = \cos 0 = 1$$

$$\frac{\partial f}{\partial z}(1,0,1) = 0$$

A dirección na que hai que moverse é na dirección $(0,1,0)$, e decir, na dirección do semieje **OY** positiva.

Derivada direccional no sentido $(1,2,2)$:

$$\mathbf{D}_{\vec{\mathbf{v}}}\mathbf{f}(1,0,1)=\nabla\mathbf{f}(1,0,1)\cdot\vec{\mathbf{v}}$$

$$\vec{\mathbf{W}}=(1,2,2)\Rightarrow\left\|\vec{\mathbf{W}}\right\|=\sqrt{1+4+4}=\sqrt{9}=3$$

$$\vec{\mathbf{v}}=\frac{1}{3}(1,2,2)$$

$$\mathbf{D}_{\vec{\mathbf{v}}}\mathbf{f}(1,0,1)=(0,1,0)\cdot\frac{1}{3}(1,2,2)=\frac{2}{3}$$

10.

$$\mathbf{f}(\mathbf{x},\mathbf{y})=9\mathbf{x}+3\mathbf{x}^2-\mathbf{x}^3-3\mathbf{y}^2+\mathbf{y}^3$$

$$\nabla\mathbf{f}(\mathbf{x},\mathbf{y})=\left(9+6\mathbf{x}-3\mathbf{x}^2,-6\mathbf{y}+3\mathbf{y}^2\right)=(0,0)\Rightarrow$$

$$\Rightarrow 9+6\mathbf{x}-3\mathbf{x}^2=0\Leftrightarrow\mathbf{x}^2-2\mathbf{x}-3=0$$

$$\Rightarrow -6\mathbf{y}+3\mathbf{y}^2=0\Rightarrow 3\mathbf{y}(\mathbf{y}-2)=0\begin{cases}\nearrow \mathbf{y}=0\\ \searrow \mathbf{y}=2\end{cases}$$

$$\mathbf{x}=\frac{2\pm\sqrt{4+12}}{2}=\frac{2\pm4}{2}=\begin{cases}\nearrow \mathbf{x}=3\\ \searrow \mathbf{x}=-1\end{cases}$$

Polo tanto, os puntos críticos da función son:

$$(3,0),(3,2),(-1,0),(-1,2)$$

11.

$$\mathbf{f}(\mathbf{x},\mathbf{y})=\mathbf{x}^2\mathbf{y} \qquad \mathbf{f}(\mathbf{x},\mathbf{y})=\mathbf{x}^2\mathbf{y} \qquad \mathbf{B}=\left\{(\mathbf{x},\mathbf{y})\in\mathbb{R}^2\,/\,\mathbf{x}^2+\mathbf{y}^2=6\right\}$$

$$\nabla\mathbf{f}(\mathbf{x},\mathbf{y})=\left(2\mathbf{xy},\mathbf{x}^2\right) \qquad \nabla\mathbf{g}(\mathbf{x},\mathbf{y})=(2\mathbf{x},4\mathbf{y})$$

$$\begin{cases} 2\mathbf{xy}=2\lambda\mathbf{x}\Rightarrow 2\mathbf{x}(\mathbf{y}-\lambda)=0 \begin{matrix}\nearrow \mathbf{x}=0 \\ \searrow \mathbf{y}=\lambda\end{matrix} \\ \mathbf{x}^2=4\lambda\mathbf{y} \\ \mathbf{x}^2+2\mathbf{y}^2=6 \end{cases}$$

Si $\mathbf{x}=0$, entonces:

$$4\lambda\mathbf{y}=0$$

$$2\mathbf{y}^2=6\Rightarrow\mathbf{y}^2=3\Rightarrow\mathbf{y}=\pm\sqrt{3}$$

Polo tanto, $\boxed{\left(0,\sqrt{3}\right),\left(0,-\sqrt{3}\right)}$

Si $\mathbf{y}=\lambda$, entonces:

$$\mathbf{x}^2=4\lambda^2\Rightarrow\mathbf{x}=\pm\sqrt{4\lambda^2}\Rightarrow\mathbf{x}=\pm2|\lambda|$$

$$\mathbf{x}^2+\mathbf{y}^2=6\Rightarrow4\lambda^2+2\lambda^2=6\Rightarrow6\lambda^2=6\mathbf{x}\Rightarrow\boxed{\lambda=\pm1}$$

Si $\lambda=1,\mathbf{y}=1,\mathbf{x}=2,\mathbf{x}=-2$ $\qquad (2,1),(-2,1)$

Si $\lambda=-1,\mathbf{y}=-1,\mathbf{x}=2,\mathbf{x}=-2$ $\qquad (2,-1),(-2,-1)$

Evaluamos ahora a función nos puntos:

$$\mathbf{f}\left(0,\sqrt{3}\right)=\mathbf{f}\left(0,-\sqrt{3}\right)=0$$

$$\mathbf{f}(2,1)=\mathbf{f}(-2,1)=4$$

$$\mathbf{f}(2,-1)=\mathbf{f}(-2,-1)=-4$$

Por tanto, 4 é o máximo absoluto de f en g e -4 é o mínimo absoluto de f en g.